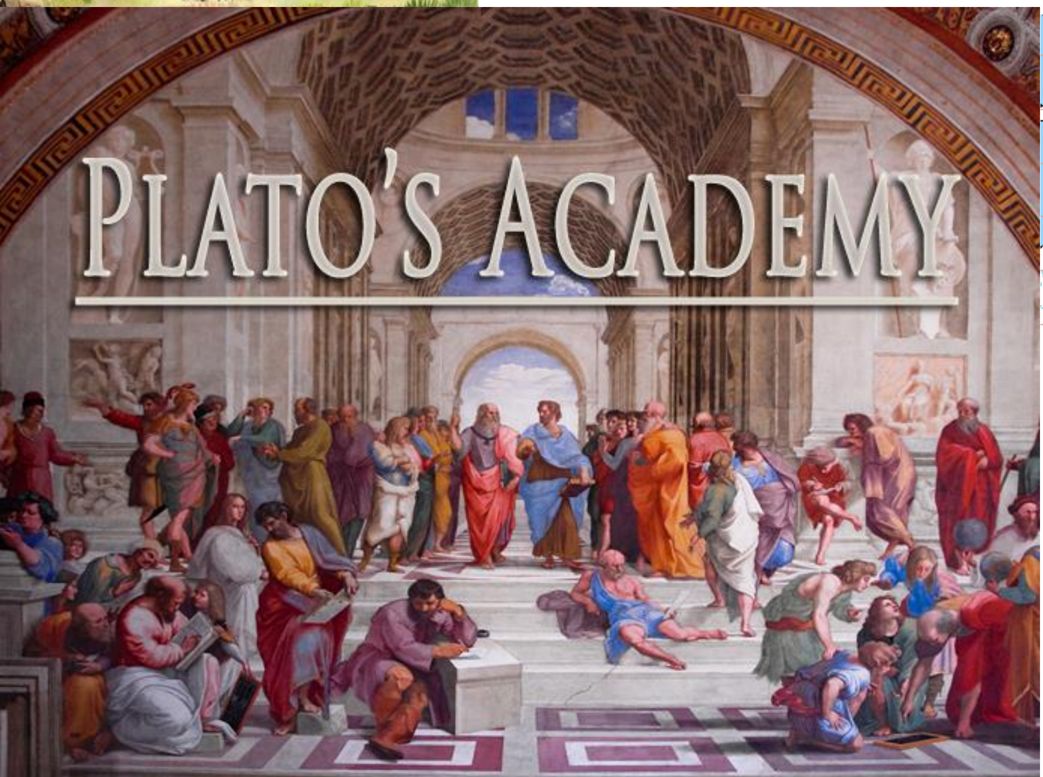
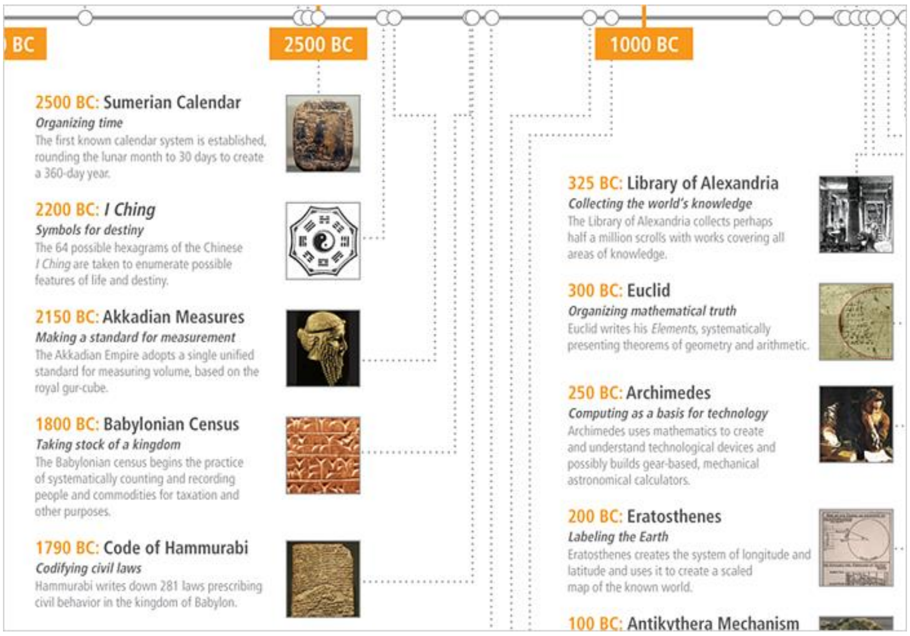
HISTORY OF MATH CONCEPTS

Essentials, Images and Memes

Sergey Finashin





Modern Mathematics having roots in ancient Egypt and Babylonia, really flourished in ancient Greece. It is remarkable in Arithmetic (Number theory) and Deductive Geometry. Mathematics written in ancient Greek was translated into Arabic, together with some mathematics of India. Mathematicians of Islamic Middle East significantly developed Algebra. Later some of this mathematics was translated into Latin and became the mathematics of Western Europe. Over a period of several hundred years, it became the mathematics of the world.

Some significant mathematics was also developed in other regions, such as China, southern India, and other places, but it had no such a great influence on the international mathematics.

The most significant for development in mathematics was giving it firm logical foundations in ancient Greece which was culminated in Euclid's *Elements*, a masterpiece establishing standards of rigorous presentation of proofs that influenced mathematics for many centuries till 19th.

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Lecture 1. Prehistory: from primitive counting to Numeral systems

Some of primitive cultures included just words for "one", "two", and "many".

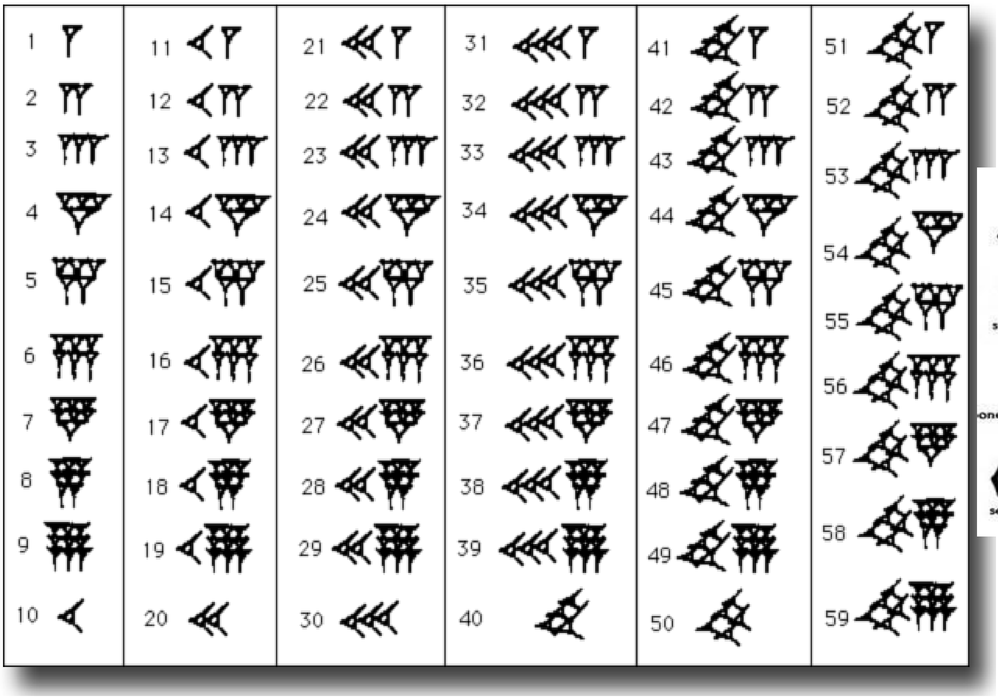
In addition to finger, the most usual tools of counting were sticks and pebbles.

The earliest (20-35 000BC) archeological artefacts used for counting are bones with a number of cuts.

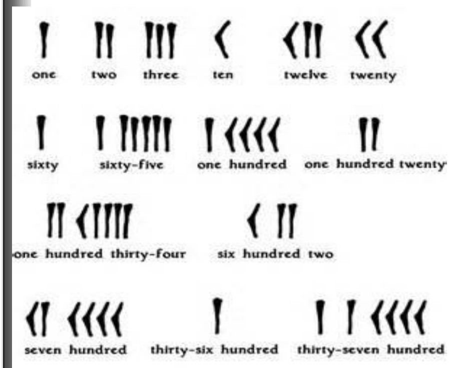

Numeral Systems

The origin of the earliest civilizations such as Sumer (in Mesopotamia), Egypt and Minoan (in Crete) goes back to 3500-4000BC. Needs of trade, city management, measurement of size, weight and time required a unified system to make calculations and represent the results. The earliest Sumerian Systems of Measures and Calendars are dated by 4000BC. Special clay tokens were invented to count sheep, days and other objects (different ones were counted with different tokens and often in a different way).

In 3000BC in the city Uruk there were more than a dozen of different counting systems in use. About this time, Abacus as a tool of calculation was invented. Later, as a writing system was developed (pressing cuneiform signs on clay tablets with a reed stylus), the Sumer sexagesimal numeral system based on powers of 60 was elaborated (do not confuse with hexadecimal system based on 16). Nowadays Sumerian system is used for time (hour, minutes, seconds) and angle measurements (360^o).



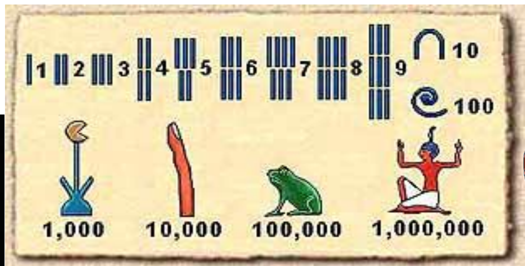
Babylonian numerals

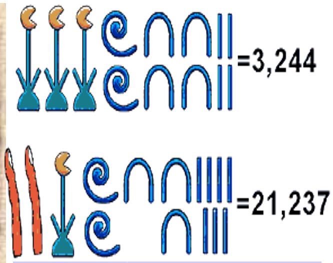


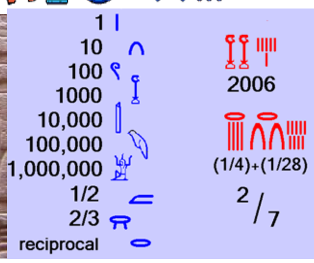
Initially a sign-value system, was gradually transformed into a place-value system. In the place-value (aka positional) systems, the same symbols are used with a different magnitude depending on their place in the number.

Egyptian numerals (2000BC)

To compare, the Egyptian numeral system (that also appeared about 2500-3000BC) is decimal: based on powers of 10. But it is a sign-value system, and so, for 10, 100, 1000, etc., different symbols are used.

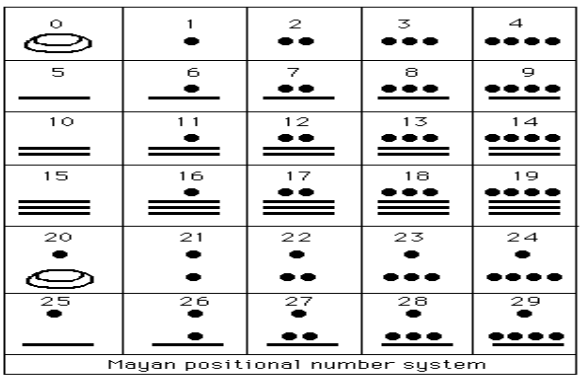


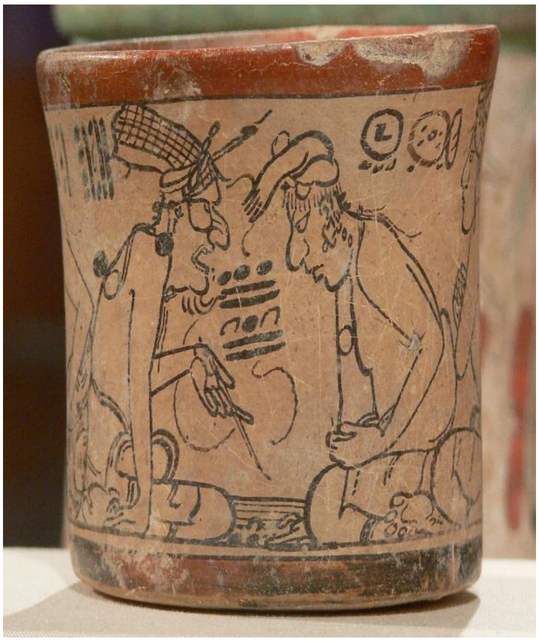




Maya numerals (650BC)

Maya developed a vigesimal (based on 20) place-value numeral system. They were the first ones who used a sign for zero (before Indians).





Greek and Roman numerals (decimal sign-value systems)

1 = I.
5 = Π or Γ from πέντε for five.
10 = Δ from δέκα for ten.
100 = H from ἑκατον for hundred.
1,000 = X from χίλιοι for one thousand.
10,000 = M from μυρίοι for ten-thousand.

I	1
II	2
III	3
IIII	4
Γ	5
ΓI	6
ΓII	7
ΓIII	8
ΓIIII	9
Δ	10
ΔΓ	15
ΔΔ	20
𐅄	50
H	100
X	1000
M	10 000

	Units	Tens	Hundreds
1	α alpha	ι iota	ρ rho
2	β beta	κ kappa	σ sigma
3	γ gamma	λ lambda	τ tau
4	δ delta	μ mu	υ upsilon
5	ε epsilon	ν nu	φ phi
6	ϝ digamma	ξ xi	χ chi
7	ζ zeta	ο omicron	ψ psi
8	η eta	π pi	ω omega
9	θ theta	ϙ koppa	ϡ sampi

In the ancient Greece several numeral system were used. In one of them known as alphabetic, or Ionic, Ionian, Milesian, and Alexandrian numerals, letters are used instead of digit.

Another numeral system called attik, or herodianic, or acrophonic, resembles the Roman numerals.

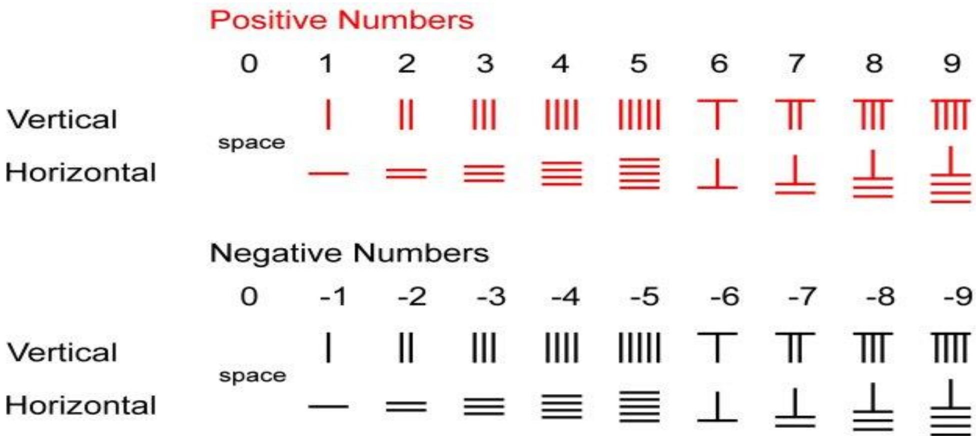
Example: 1982 = X𐅅HHHH𐅄ΔΔΔII = MCM LXXXII

The acrophonic numerals in comparison to the Roman numeral system.

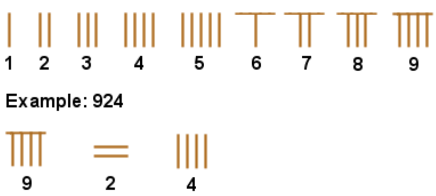
I	Π	Δ	𐅄	H	𐅅	X	𐅆	M
			5 × 10		5 × 100		5 × 1000	10000
1	5	10	50	100	500	1000	1000 × 5	1000 × 10
I	V	X	L	C	D	M		

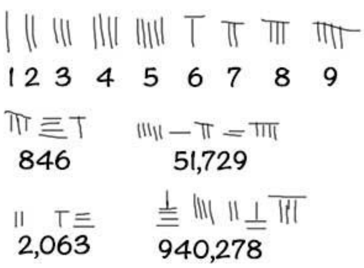
Stigma (ϛ) is a ligature of the Greek letters sigma (Σ) and tau (T), which was used in writing Greek for the number 6. In this function, it is a continuation of the old letter digamma, F, which was conflated with the σ-τ ligature in the Middle Ages.

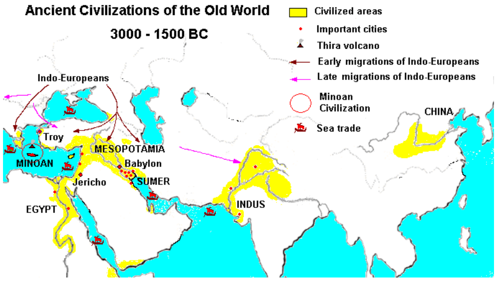
Chinese rod numbers (decimal place-value system 1300BC)

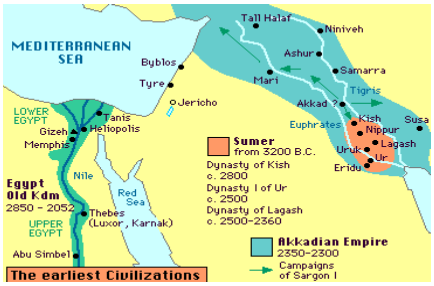


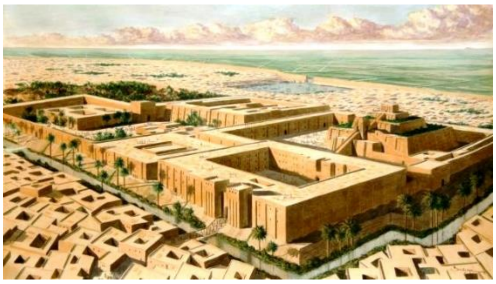
In addition to a hieroglyphic sign-value numeral system in ancient China, Rod numbers were invented: they existed in vertical and horizontal forms. In writing they were alternating: vertical form was used for units, hundreds, tens of thousands, etc., while horizontal rods were used for tens, thousands, etc. Chinese developed (100BC) negative numbers and distinguished them from positive ones by color.











Lecture 2. Archaic Mathematics in Mesopotamia (Babylonia) and Egypt

Babylonian Mathematics: not much of geometry, but amazing arithmetic and algebra

Babylonian mathematics used pre-calculated clay tablets in cuneiform script to assist with arithmetic. For example, two tablets found at Senkerah on the Euphrates in 1854, dating from 2000 BC, give lists of the squares of numbers up to 59 and the cubes of numbers up to 32. Together with the formulae

$$ab = \frac{(a+b)^2 - a^2 - b^2}{2} \qquad ab = \frac{(a+b)^2 - (a-b)^2}{4}.$$

the tables of squares were used for multiplication. For division a table of reciprocals was used together with the formula $\frac{a}{b} = a \times \frac{1}{b}$

Numbers whose only prime factors are 2, 3 or 5 (known as 5-smooth or regular numbers) have finite reciprocals in sexagesimal notation, and tables with lists of these reciprocals have been found. To compute 1/13 or to divide a number by 13 the Babylonians would use an approximation such as

$$\frac{1}{13} = \frac{7}{91} = 7 \times \frac{1}{91} \approx 7 \times \frac{1}{90} = 7 \times \frac{40}{3600} = \frac{280}{3600} = \frac{4}{60} + \frac{40}{3600}.$$

To solve a quadratic equation $x^2 + bx = c$ the standard quadratic formula was used $x = -\frac{b}{2} + \sqrt{\left(\frac{b}{2}\right)^2 + c}$ with the tables of squares in reverse to find square roots.

Here *c* was always positive and only the positive root was considered “meaningful”. Problems of this type included finding the dimensions of a rectangle given its area and the amount by which the length exceeds the width.

The tables for finding square and cubic root were up to 3 sexagesimals (5 decimals). To improve an approximation $x_1^2 \sim a$ the formula $x_2=1/2(x_1+a/x_1)$ was used. For example, to find the square root of 2 one can take $x_1=1.5$ as the first approximation, $x_1^2 \sim 2$. Then $x_2=1/2(x_1+2/x_1) \sim 1/2(1.5+1.3)=1.4$ is a better approximation.

Other tables did exist to solve a system $x+y=p$, $xy=q$ that is equivalent to $x^2-px+q=0$.

Tables of values of $n^3 + n^2$ were used to solve certain cubic equations, like $ax^3 + bx^2 = c.$

Multiplying the equation by a^2 and dividing by b^3 and letting $y = ax/b$ we obtains

$$\left(\frac{ax}{b}\right)^3 + \left(\frac{ax}{b}\right)^2 = \frac{ca^2}{b^3}. \qquad y^3 + y^2 = \frac{ca^2}{b^3}$$

where y can be found now from the table.

Babylonian algebra was not symbolic, but it was rhetoric: instead of symbols for unknown and signs just words were used, for example, an equation $x+1=2$ was expressed as “a thing plus one equals two”.

For finding the length of a circle and the area of a disc an approximate value $\pi \sim 25/8=3.125$ was known, although an approximation $\pi \sim 3$ was also often used.

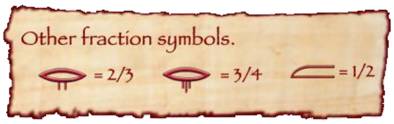
The Plimpton 322 tablet (1800 BC) in Plimpton collection at Columbia University. It contains a list of Pythagorean triples, i.e., integers (a,b,c) such that $a^2+b^2=c^2$. It seems that a general formula for such triples was known, although no direct evidence of this was ever found.

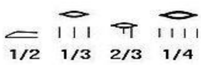
Problems related to growth of loans were well-developed.

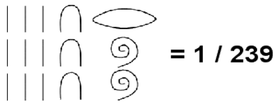
Astronomical calculations allowing to predict motion of planets were developed at a high level.

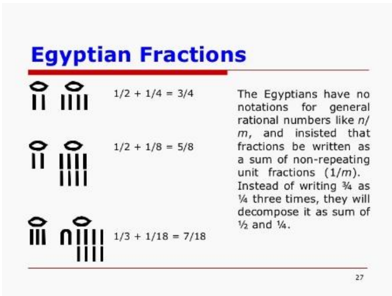
During the Archaic period of Greece (800-500BC) Babylon was famous as the place of studies.

Egyptian mathematics: unit fractions, more geometry, but less algebra









Rhind (or Ahmes) mathematical papyrus (1650 BC) in British Museum, 6m length. It was found during illegal excavation and sold in Egypt to Scottish antiquarian Rhind in 1858. It is a problem book that was copied by scribe Ahmes from an older papyrus dated by 1800-2000BC.

There are 87 problems with solutions in arithmetic, algebra and geometry. The most of arithmetical problems are related to the unit Egyptian fractions and involve in particular finding least common multiples of denominators and decomposition of 2/n into unit fractions. A dozen of problems are related to linear equations, like x + x/3 + x/4 = 2 (in modern notation) and a few more are devoted to arithmetic and geometric progressions.

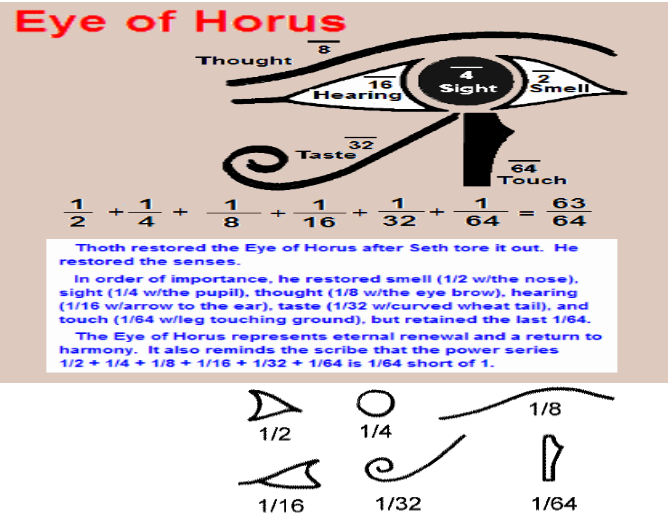
The geometric problem include finding areas of rectangles, triangles and trapezoids, volumes of cylindrical and rectangular based granaries, and the slopes of pyramids.

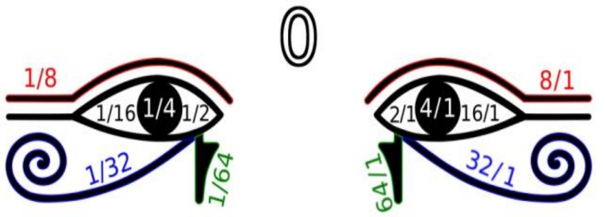
The volume V of a cylindrical granary of a diameter d and height h was calculated by formula

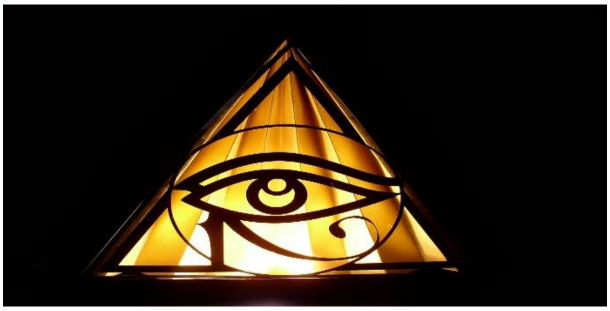
$V=[(1-1/9)d]^2h$ or in modern notation $V=(8/9)^2d^2h=(256/81)r^2h$ where d = 2r, and the quotient 256/81 approximates the value π ~ 3.1605.

Another famous Moscow Mathematical Papyrus (1800BC) contains 25 problems, and some of them are of a different kind: on finding the area of surfaces such as a hemisphere and a truncated pyramid.

From these and a few more papyri one may conclude that Egyptians knew arithmetic, geometric and harmonic means. They had a concept of perfect and prime numbers, and used sieve of Eratosthenes.

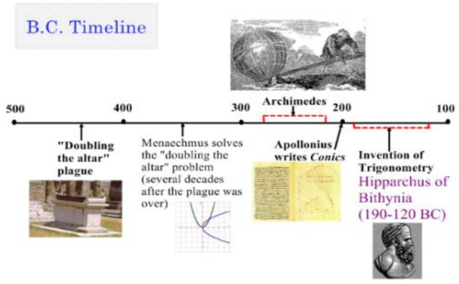






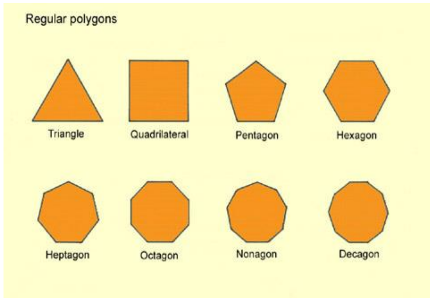
Questions to Lectures 1-2:

- What did China, India, Egypt and Babylon have in common?
- What were the earliest causes for the creation of mathematics?
- Why were so many different bases (i.e. 2, 3, 5, 10, 20, 60) used?
- Was early mathematics recreational, theoretical, applied, or what?
- Was the idea of proof or justification used or needed?
- Why conic sections were never considered?
- How are nonlinear equations considered, solved? What do the Egyptians do? What do the Babylonians do?
- What was the relation between the exact and the approximate? Was the distinction clearly understood?

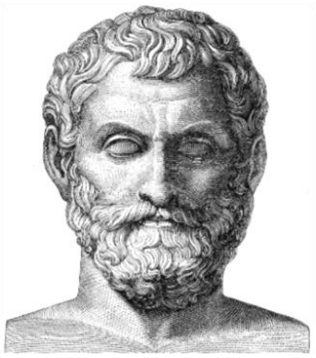


Lecture 3. Birth of mathematics as a deductive science in Greece: Thales and Pythagoras

- Archaic Period 776 BC (The first Olympic games)/500BC (Beginning of Persian Wars)
- Classical Period 500 /323BC (death of Alexander)
- Early Hellenistic Period 323BC/146AD
- Late Hellenistic Period 146/500AD



Words: μαθεμα - knowledge, αριθμος - number, γεωμετρια - geometry.

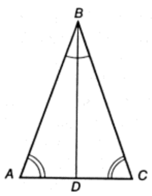


Thales of Miletus (Θαλης) 624-546 BC the first philosopher and mathematician in Greek tradition, one of seven Sages of Greece, founder of Milesian natural philosophy school

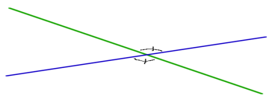
Recognized as an initiator of the scientific revolution: rejected mythological explanation and searched for a scientific one. He was interested in physical world and for application of knowledge to it.

Thales introduced a concept of proof as a necessary part of mathematical knowledge (proofs did not look that important in previous mathematics commonly viewed rather as a collection of facts and practices in calculation). So, he distinguished mathematics as a science from application of it to engineering and other purposes. He separated in particular arithmetic as a science about numbers from the art of computation that he called logistic.

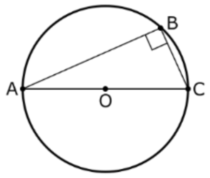
He considered separately two kinds of numbers: "arithmetical" natural numbers and "geometric" numbers that are results of measurements (say, length) with a scale.



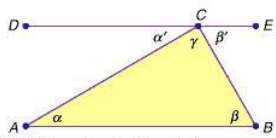
Thales introduced the idea of "construction" problems in geometry, in which only a compass and straightedge can be used. Giving a solution to the problem of bisection of angle, he stated the problem of trisection.



Some theorems usually attributed to Thales:



1) on isosceles triangles: two sides are equal if and only if the angles are equal
2) the sum of angles of a triangle is 180°
3) opposite angles between two lines are equal
4) similar triangles (with the same angles) have proportional sides
5) if AC is a diameter, then the angle at B is a right angle.



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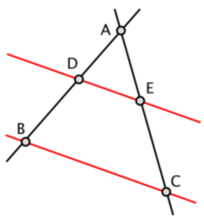
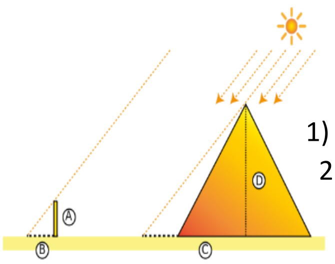
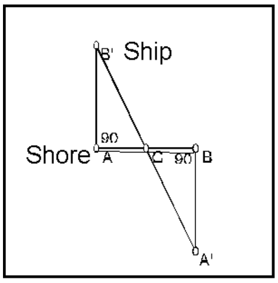
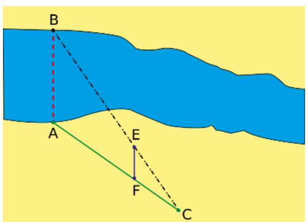


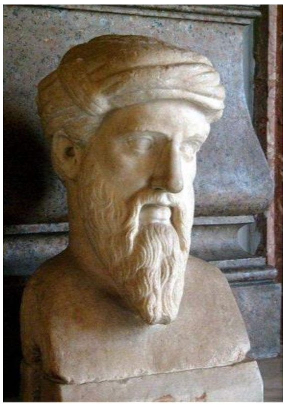


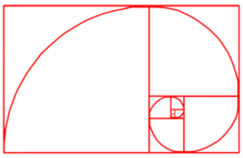
Some famous applications of his knowledge to practical needs:

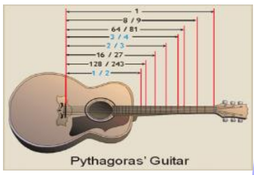
1) How to measure the height of a pyramid?
2) How to find the distance from a ship to a shore?
3) How to measure the width of a river?



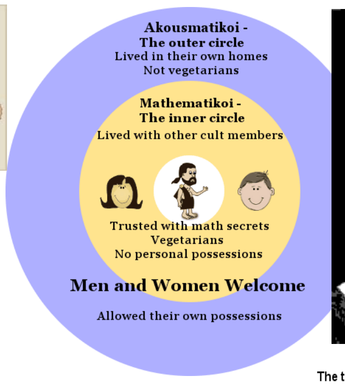


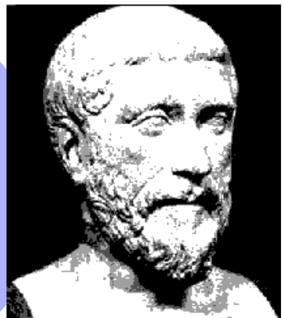






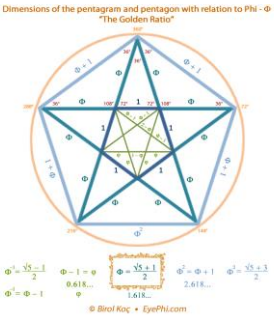
Pythagoras from Samos (Πυθαγορας) 580-500 BC



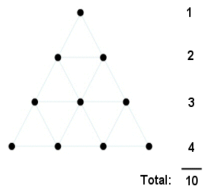


Pythagoras

After leaving Samos, where Pythagoras had a conflict with its tyrant, he settled in Croton and established a school, a kind of esoteric society and brotherhood with somewhat strict rules of life, called "Mathematikoi". The following achievements are attributed to Pythagoras or his followers:



The tetractys, an equilateral triangular figure consisting of 10 points arranged in four rows of 1, 2, 3 and 4, was both a mathematical idea and a metaphysical symbol for the Pythagoreans.



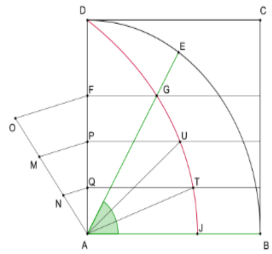
1) "Principle of the world harmony"; Pythagorean tuning, "music of spheres"
2) Theory of primes, polygonal numbers, squares and ratios of integers and other magnitudes
3) Irrationality of square root of 2, etc. (some attribute to his students, e.g., Hippasus)
4) Studying of the Golden Ratio and Pentagram (symbol of Pythagoreans) a sign of math perfection
5) The problem of construction regular polygons (pentagon and some others were constructed)
6) Geometric algebra: solving equations like $a(a-x)=x^2$ geometrically
7) Regular solids (Pythagoras himself knew possibly only three of them)
8) Doctrine of quadrature: "to understand the area means to construct a square by means of compass and straightedge"; the problem of Quadrature of circle
9) "Pythagoras theorem" with numerous proofs, "Pythagoras triples" (although known in Babylon)
10) Four Pythagorean Means, their geometric presentation and comparison
11) Astronomy: spherical shape of the Earth, Sun as the center of the world, Venus as a morming and evening star (it was considered as two different ones)
12) Medicine: brain is a locus of the soul

Example: Prove $\sqrt{2}$ is irrational.

Proof: Assume $\sqrt{2}$ is rational. That means that $\sqrt{2}=\frac{a}{b}$, where a and b are relatively prime integers.

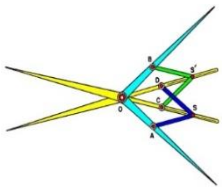
$$\sqrt{2}=\frac{a}{b}$$
$$2=\frac{a^2}{b^2}$$
$$2b^2=a^2$$

Since $2b^2=a^2$, 2 must be a factor of a^2. 2 is thus a factor of a, so it turns out that 4 is a factor of a^2. Since 4 is a factor of a^2 and $2b^2=a^2$, it follows that 4 is a factor of $2b^2$. Hence 2 must be a factor of b^2. This means that 2 must be a factor of b. 2 is thus a factor of both a and b, so a and b are not relatively prime. This contradicts the assumption that $\sqrt{2}$ is rational. By contradiction, $\sqrt{2}$ is irrational.

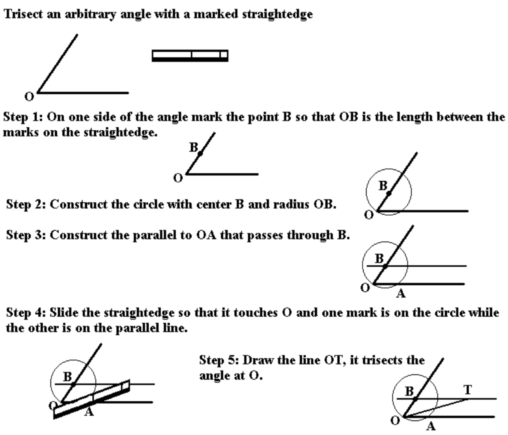


"The Pythagoreans, who were the first to take up mathematics, not only advanced this subject, but saturated with it, they fancied that the principles of mathematics were the principles of all things."
ARISTOTLE, 384 – 322 BC *Metaphysica*

Great Construction problems of Ancient Greece:



1. Trisecting the angle (stated possibly by Thales)



2. Squaring the circle (stated possibly by Pythagoras)
3. Doubling the cube (attributed to Plato)

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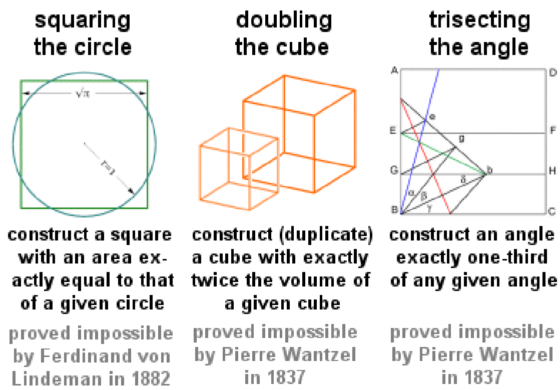
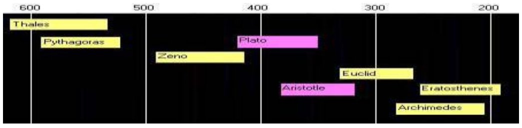
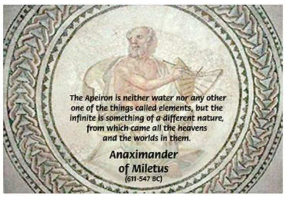
4. Construction of a regular n-gon (attributed to Pythagoras)

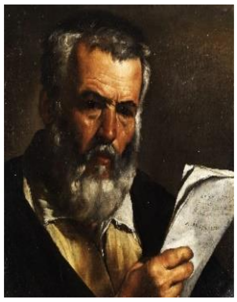


Anaximenes
585- 528BC



Lecure 4. Important developments of ideas in the classical period





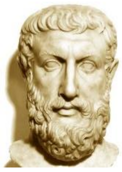
Anaximander
610-546 BC

Heraclites

Milesian school (of Miletus): founded by Thales. His student Anaximander: claimed *apeiron* as the primary element, introduced gnomon, created a map of the world. For Anaximenes air was primary.

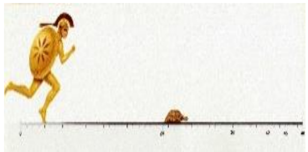
Heraclites of Ephesus 535-475BC known as "weeping philosopher" Πάντα ῥεῖ

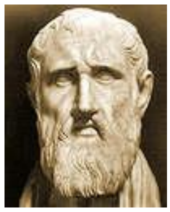
"Panta rei" (everything flows), "No man ever steps in the same river twice"; "The path up and down are one and the same" (on the unity of the opposites); "All entities come to be in accordance with Logos" (here Logos is a word, reason, plan, or formula).



Parmenides

Eleatic school (of Elea): founded by Parmenides (540-?BC) Disputed with Heraclites and claimed that "anything that changes cannot be real" and that "truth cannot be known through perception, only Logos shows truth of the world"; "You say there *is* a void; therefore the void is not nothing; therefore there is not the void."

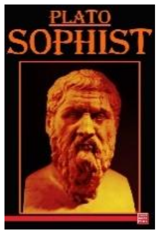




Zeno

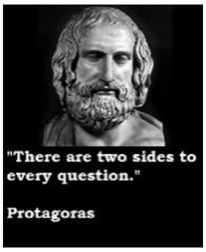
Zeno of Elea (Ζηνων) 490-430BC student of Parmenides, stated aporias (paradoxes) such as "Achill and tortoise", "Arrow", etc.

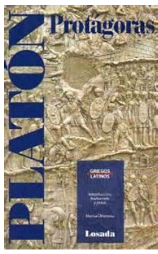
Democritus 460-370BC "laughing philosopher" born in Abdera also some links him with the Milesian school. With his teacher Leucippus proposed an atomic theory as an answer to the aporias of Zeno.

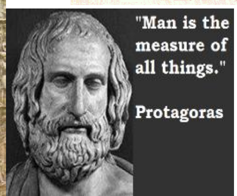


Sophists (Protagoras, Gorgias, Prodicus, Hippias, etc.) were a category of teachers (mostly in 500-3500 BC) who specialized in using the techniques of philosophy, rhetoric (skill of public speaking) and dialectic (skill to argue in a dialogue by showing contradictions in opponent's viewpoint) for the purpose of teaching arête (excellence, or virtue) predominantly to young statesmen and nobility.

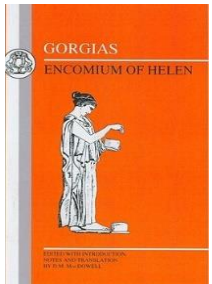
Protagoras 490-420 BC: Taught to care about proper meaning of words (orthoepeia). "Man is the measure of all things"; "Concerning the gods, I have no means of knowing whether they exist or not, or what sort they may be, because of obscurity of the subject, and the brevity of human life." Athenians expelled him from the city, and his books were collected and burned on the market place.



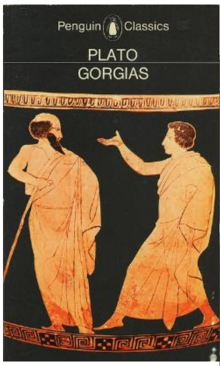


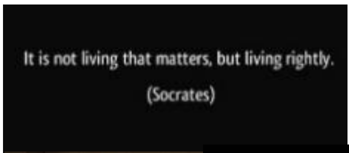


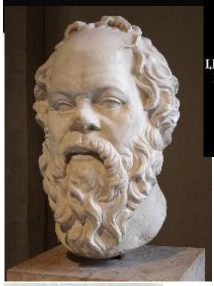
Gorgias 485-380: performed oratory, like "Encomium of Helen"; ironic parody "On the nature of non-existent": 1) *Nothing exists*. 2) *Even if something exists, nothing can be known about it.* 3) *Even if something can be known about it, this knowledge cannot be communicated to the others.* 4) *Even if it can be communicated, it cannot be understood*. True objectivity is impossible. "*How can anyone communicate an idea of color by means of words, since ear does not hear colors but only sounds?*" Love to paradoxologia.

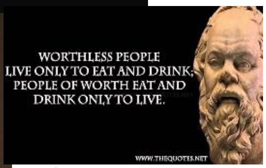


Gorgias

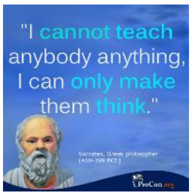


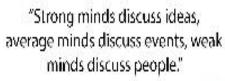


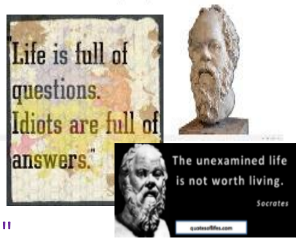


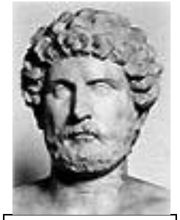


Socrates 470-399BC credited as one of the founders of Western philosophy. "Socratic Method" of teaching (possibly invented by Protagoras) through a dialogue is demonstrated in the book of his student, Plato. Proposed to switch attention to a human and his thinking from nature of the physical world. "I know that I know nothing."



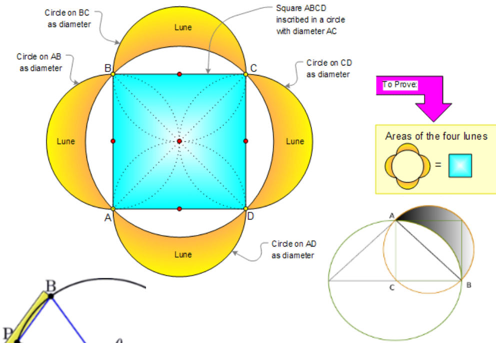


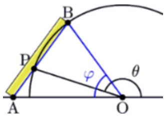




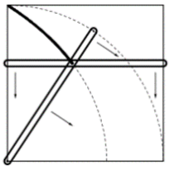
Hippocrates

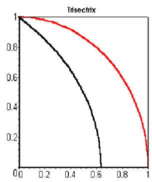
Hyppocrates of Chios 470-410BC (do not confuse with Hippocrates of Kos, father of Western Medicine) was Pythagorean, but then quitted. He has written the first "Elements" (Euclid 3-4) and discovered quadrature of Lunes as a partial quadrature of circle. He stated the principle to avoid neusis constructions (otherwise, trisection of an angle would be possible).

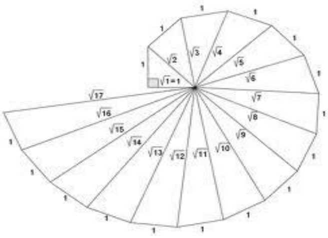




Hippias of Elis 460-400BC a sophist lecturing on poetry, grammar, history, politics, and math Invented Trisectrix (known also as Quadratrix after Dinostratus 390-320BC used it for squaring circle).



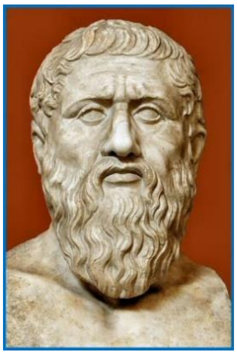




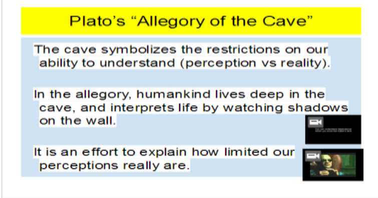
Theodorus of Cyrene 465-398BC student of Protagoras and tutor of Plato. Spiral made of right triangles whose hypotenuses are square roots from 2 to 17

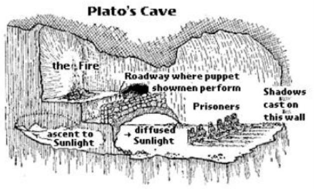
Greek Mathematicians with their Home-Cities

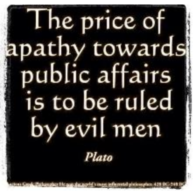
• Abdera: Democritus • Alexandria: Apollonius, Aristarchus, Diophantus, Eratosthenes, Euclid, Hypatia, Hypsicles, Heron, Menelaus, Pappus, Ptolemy, Theon • Amisus: Dionysodorus • Antinopolis: Serenus • Apameia: Posidonius • Athens: Aristotle, Plato, Ptolemy, Socrates, Theaetetus • Byzantium (Constantinople): Philon, Proclus • Chalcedon: Proclus, Xenocrates • Chalcis: Iamblichus	• Chios: Hippocrates, Oenopides • Clazomenae: Anaxagoras • Cnidus: Eudoxus • Croton: Philolaus, Pythagoras • Cyrene: Eratosthenes, Nicoteles, Synesius, Theodorus • Cyzicus: Callippus • Elea: Parmenides, Zeno • Elis: Hippias • Gerasa: Nichmachus • Larissa: Dominus • Miletus: Anaximander, Anaximenes, Isidorus, Thales	• Nicaea: Hipparchus, Sporus, Theodosius • Paros: Thymaridas • Perga: Apollonius • Pergamum: Apollonius • Rhodes: Eudemus, Geminus, Posidonius • Rome: Boethius • Samos: Aristarchus, Conon, Pythagoras • Smyrna: Theon • Stagira: Aristotle • Syene: Eratosthenes • Syracuse: Archimedes • Tarentum: Archytas, Pythagoras • Thasos: Leodamas • Tyre: Marinus, Porphyrius



Lecture 5. Academy of Plato and his circle. Aristotle and his Logic

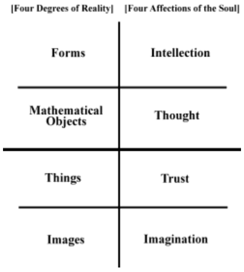




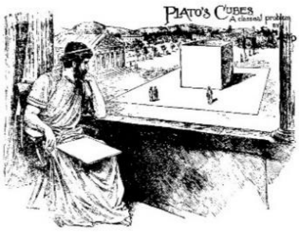


Plato 428-348 philosopher and mathematician, the author of Dialogues (the first original philosophical text that came to us almost untouched)

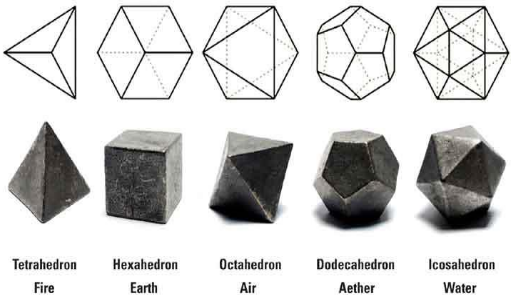
Established the Western called the Academy in a park of Athens, the first higher education center in World. The object of interest are *pure forms* or *ideas* (of a human) archetypes



"Nobody without arithmetic, can be considered educated learning five disciplines of math: plane geometry, solid geometry, astronomy and harmony".



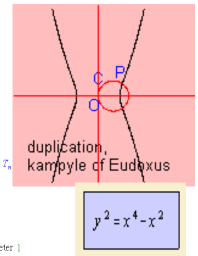
Associated an element with each regular solid: fire for tetrahedron, Earth for cube, air for octahedron, water for icosahedron and ether or prana of the whole universe for dodecahedron.

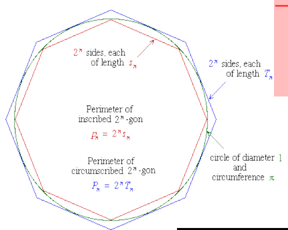


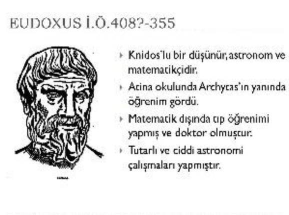
Platonic Solids as the Classic Elements

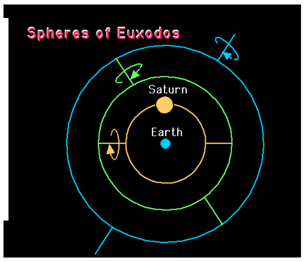
Legend about Delian Problem (Doubling of a cube)

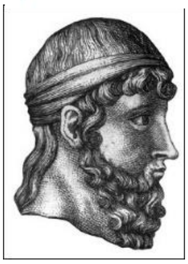
Eudoxus of Cnidus (now Datcha) 410-355 one of the greatest ancient mathematicians, astronomer, studied at Academy of Plato for 2 months, but had no money to continue He. Studied irrationals, developed a theory of proportions which was taken by Euclid into Elements 5, "two magnitudes are comparable is a multiple of one is greater than the other". He invented Method of Exhaustion (a form of integration) that was later advanced by Archimedes, created a School and criticized Plato, who was his rival. Eudoxus constructed an observatory, proposed a planetary model, the first astronomer to map stars.



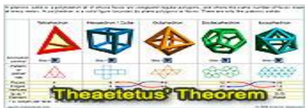


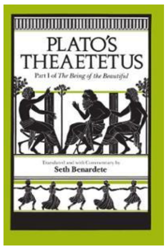




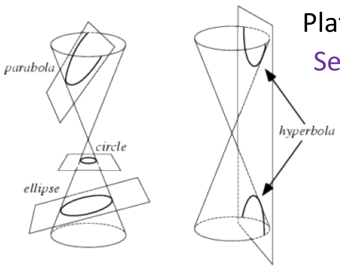


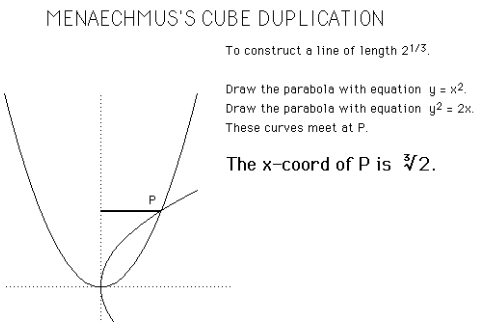
Theaetetus 417-369BC studied in Academy, a friend of Plato and a character in "Dialogues"Theory of irrational (incommensurable) magnitudes (taken to Euclid's Elements 10) Developped construction of regular solids.

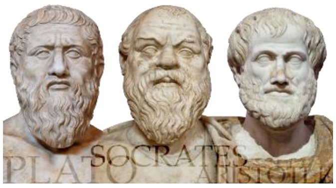


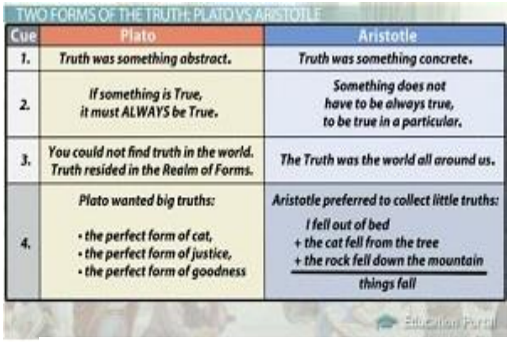


Menaechmus 380-320BC brother of Dinostratus, student of Eudoxus, friend of Plato, tutor of Alexander the Great. The first person who studied the Conic Sections and used them for solution of the doubling cube problem.





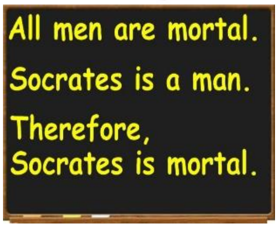


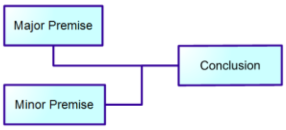


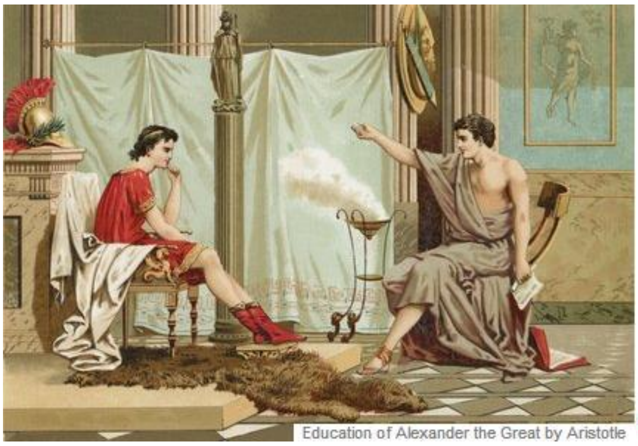
TWO FORMS OF THE TRUTH: PLATO VS ARISTOTLE

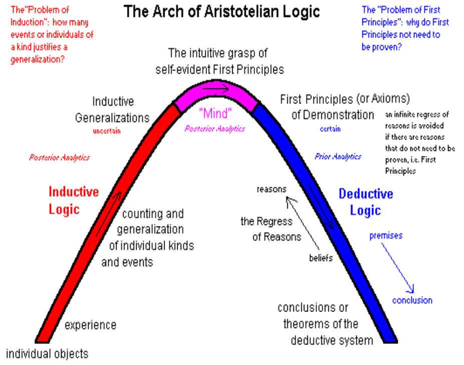
Cue	Plato	Aristotle
1.	Truth was something abstract.	Truth was something concrete.
2.	If something is True, it must ALWAYS be True.	Something does not have to be always true, to be true in a particular.
3.	You could not find truth in the world. Truth resided in the Realm of Forms.	The Truth was the world all around us.
4.	Plato wanted big truths: • the perfect form of cat, • the perfect form of justice, • the perfect form of goodness	Aristotle preferred to collect little truths: I fell out of bed + the cat fell from the tree + the rock fell down the mountain = things fall

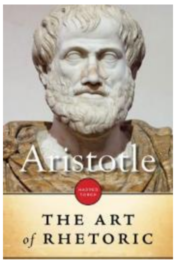
Aristotle (Αριστοτελης) 384-322BC philosopher and scientist, student of Plato, tutor of Alexander the Great After death of Plato quitted from Academy and founded Lyceum in Athens, in 335BC. Aristotle was the first who analyzed the Formal Logic and developed its "grammar", notion of syllogism. "Reason rather than observation at the center of scientific effort".

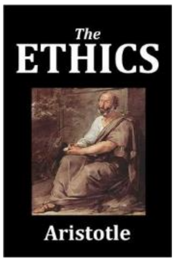


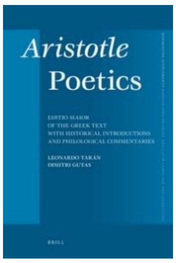


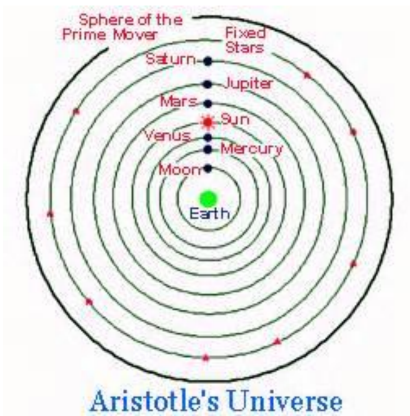

Education of Alexander the Great by Aristotle

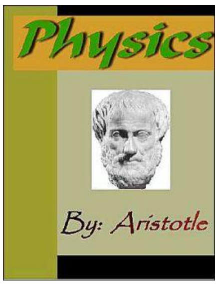


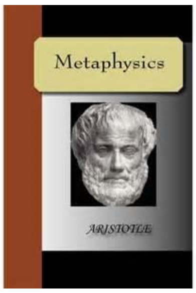


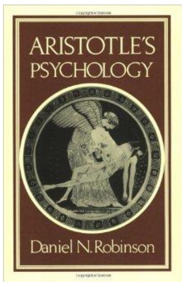


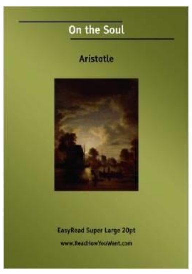


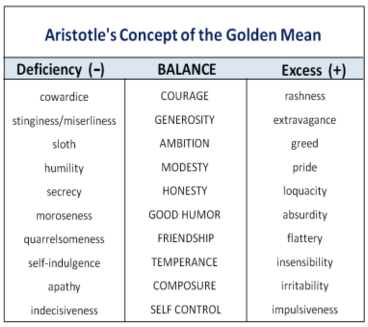






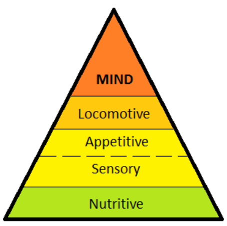




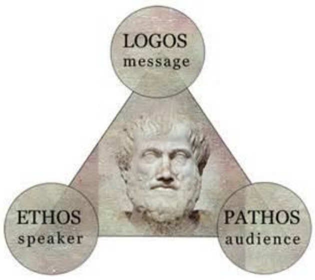


Aristotle's Concept of the Golden Mean

Deficiency (-)	BALANCE	Excess (+)
cowardice	COURAGE	rashness
stinginess/miserliness	GENEROSITY	extravagance
sloth	AMBITION	greed
humility	MODESTY	pride
secrecy	HONESTY	loquacity
moroseness	GOOD HUMOR	absurdity
quarrelsomeness	FRIENDSHIP	flattery
self-indulgence	TEMPERANCE	insensibility
apathy	COMPOSURE	irritability
indecisiveness	SELF CONTROL	impulsiveness



Aristotle's hierarchy of soul functions
(from *On the Soul*, book II, chapter 3, compiled by David Huisjen)



Lectures 6. Hellenistic Golden Age, Euclid of Alexandria

Mouseion (or Musaeum) at Alexandria, included the Library of Alexandria, was a research institution similar to modern universities founded in the end of middle of 3d century BC. In addition to the library, it included rooms for the study of astronomy, anatomy, and even a zoo of exotic animals. The classical thinkers who studied, wrote, and experimented at the Musaeum worked in mathematics, astronomy, physics, geometry, engineering, geography, physiology and medicine. The library included about half million of papyri. Hellenistic Golden Age includes primarily Euclid, Archimedes and Apollonius.

Euclid (Ευκλειδης) of Alexandria 323–283 BC "father of geometry", the author of

Elements, one of the most influential works in the history of mathematics, serving as the main textbook for teaching mathematics (especially geometry) from the time of its publication until the late 19th or early 20th century. In the *Elements*, Euclid deduced the principles of what is now called Euclidean geometry from a small set of axioms. 13 books of *Elements* the whole math knowledge of that time was summarized. The first 6 books of *Elements* are devoted to Plane Geometry, next 3 to arithmetic, and last 3 to spatial geometry.

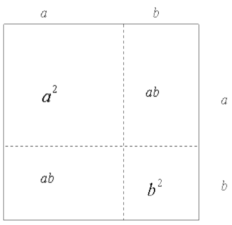
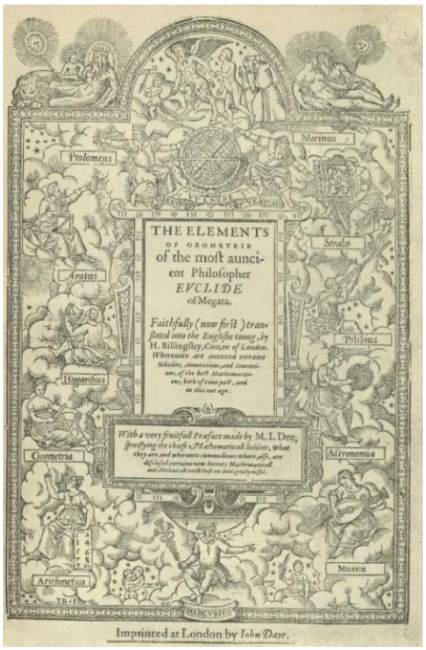
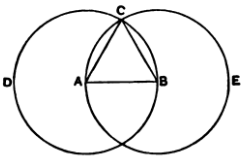


Figure 3

1. Basis plane Geometry: angles, areas (up to Pythagoras theorem)
2. Geometric Algebra (Pythagoras)
3. Circles, inscribed angles, tangents (Thales, Hippocrates)
4. Incircle, circumcircle, construction of regular polygons (with 4,5,6,15 sides).
5. Proportions of magnitudes (Eudoxus), arithmetical and geometric Mean
6. Proportions in Geometry: similar figures (Theon, Pythagoras)
7. Arithmetic: divisibility, primes, Euclid's algorithm for g.c.d. and l.c.m., prime decomposition
8. Proportions in arithmetic: geometric sequences
9. Infiniteness of number of primes, sum of geometric series, a formula for even perfect numbers
10. Theory of irrationals and method of exhaustion (based on Eudoxus)
11. Extension of the results of Books 1-6 to space: angles, perpendicularity, volumes.
12. Volumes of cones, pyramids, cylinders and spheres (Theaetetus)
13. Five Platonic solids, their size, proof that there is no other regular solids (Theaetetus).

Other books of Euclid: *Data*, *On division of Figures*, *Catoptrics, Optics, Phaenomena, Conics, Porisms*, etc.





Lecture 7. Elements and their role in the history of Mathematics

Euclid's Axioms (or common notions)

1. Things that are equal to the same thing are also equal to one another.
2. If equals are added to equals, the wholes are equal.
3. If equals are subtracted from equals, the remainders are equal.
4. Things that coincide with one another are equal to one another.
5. The whole is greater than the part.

THE

ELEMENTS OF EUCLID.

BOOK I.

DEFINITIONS.

I.
A POINT is that which hath no parts, or which hath no magnitude.

II.
A line is length without breadth.

III.
The extremities of a line are points.

IV.
A straight line is that which lies evenly between its extreme points.

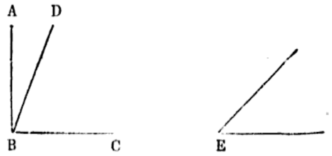
V.
A superficies is that which hath only length and breadth.

VI.
The extremities of a superficies are lines.

VII.
A plane superficies is that in which any two points being taken,* the straight line between them lies wholly in that superficies.

VIII.
"A plane angle is the inclination of two lines to one another* in a plane, which meet together, but are not in the same direction."

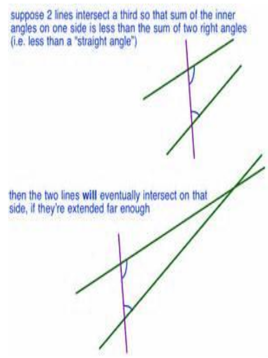
IX.
A plane rectilineal angle is the inclination of two straight lines to one another, which meet together, but are not in the same straight line.

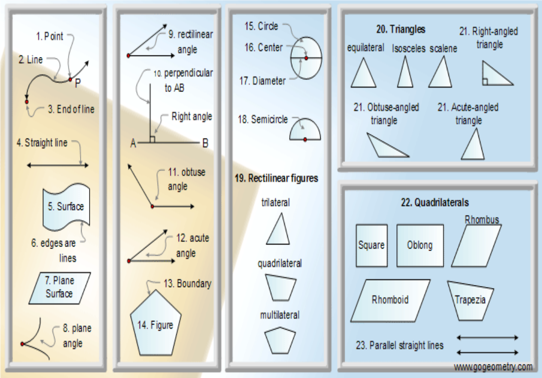


* See Notes.

The structure of Propositions:

1. Enunciation (statement of the proposition).
2. Setting-out (gives a figure and denote its elements by letters).
3. Specification (restates the general statement in terms of this figure).
4. Construction (extends the figure with new elements needed for the proof).
5. Proof (using previous Propositions).
6. Conclusion (connects the proof to the initial claim in the enunciation).



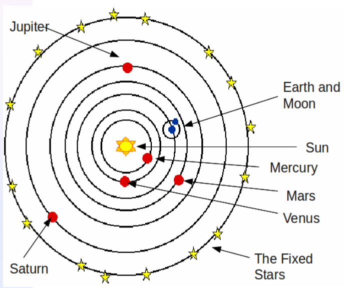


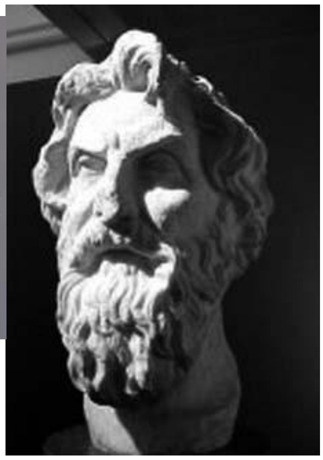
Book #	# Propositions	# Definitions	#Axioms	#Postulates
1	48	23	5	5
2	14	2		
3	37	11		
4	16	7		
5	25	18		
6	33	4		
7	39	22		
8	27	0		
9	36	0		
10	115	16		
11	39	28		
12	18	0		
13	18	0		
TOTAL:	465	131	5	5

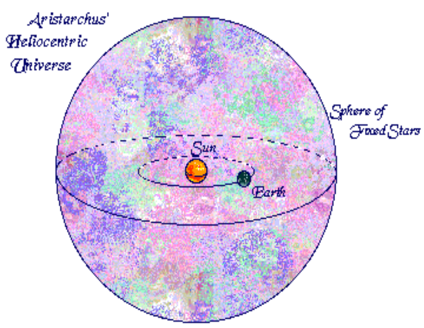
Aristarchus of Samos

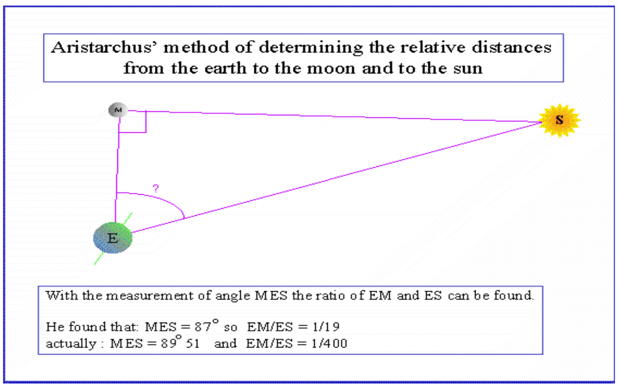
310-230BC astronomer and mathematician

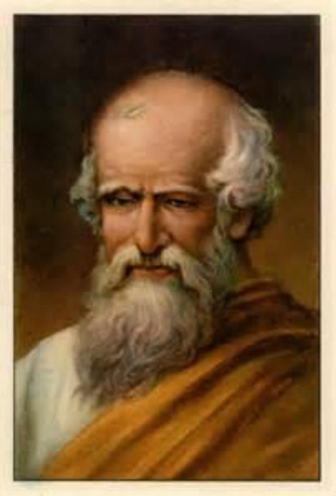
Estimated the size of Moon, found that the Earth revolves around the Sun and the Moon around the Earth.

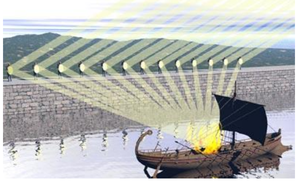








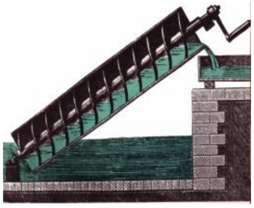


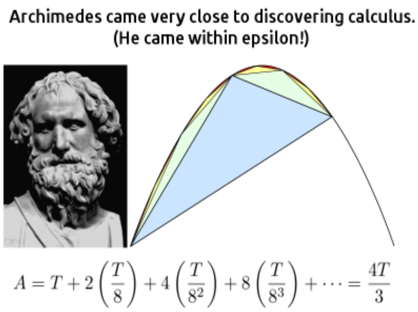


Lecture 8. Archimedes, Eratosthenes

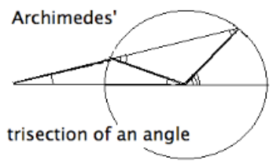
Archimedes (Αρχιμεδης) 287-212BC

mathematician, physicist, engineer, astronomer, inventor
regarded as one of the leading (in fact, the greatest) scientists in classical antiquity.

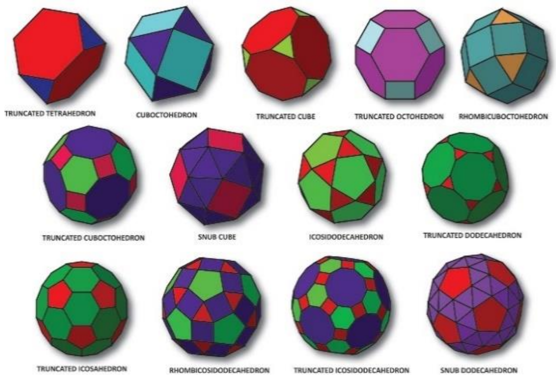


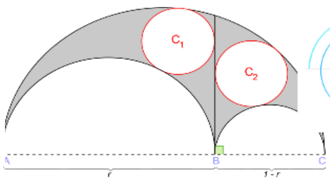


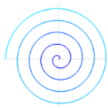
1. Concepts of infinitesimals and the method of exhaustion were developed to derive and rigorously prove a range of geometrical theorems, including the area of a circle, the area of a parabolic sector, its centroid, , the surface area and volume of a sphere and other rotational solids, parabolic and hyperbolic conoids

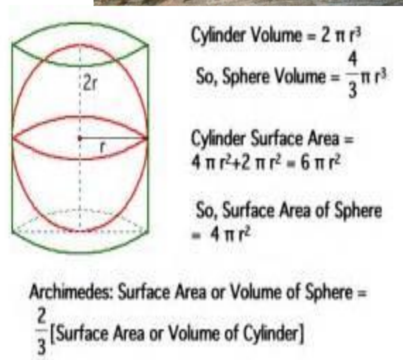


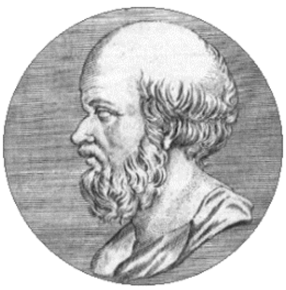
2. Proved an approximation $3\frac{10}{71} < \pi < 3\frac{1}{7}$ using inscribed and circumscribed polygons
3. Creating a system for expressing very large numbers (Sand Reckoner)
4. Applying mathematics to physical phenomena, founding hydrostatics (Archimedes' principle concerning the buoyant force) and statics (the principle of the lever).
5. Designing innovative machines, such as his screw pump, compound pulleys, and defensive war machines (Claw of Archimedes, Heat Ray, etc.) to protect his native Syracuse from invasion.
6. Investigating the Archimedean spiral
7. 13 Archimedean (semiregular) solids
8. Archimedes' twin-circles in arbelos





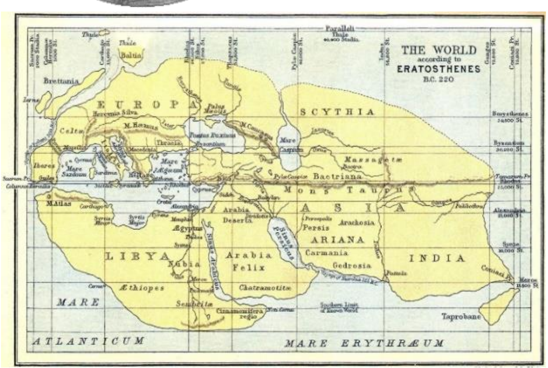


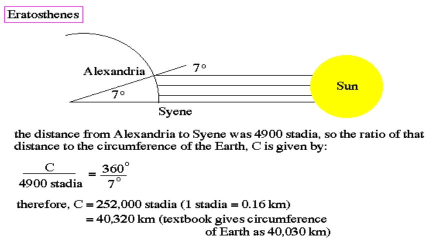




Eratosthenes of Cyrene 276-194BC

a Greek mathematician, geographer, poet, astronomer, and music theorist. He was a man of learning and became the chief librarian at the Library of Alexandria. He invented the discipline of geography, including the terminology used today. He is the founder of scientific chronology and revised the dates of the main political events from the conquest of Troy. In Math his most famous invention is Sieve of Eratosthenes for prime numbers.





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31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~60~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
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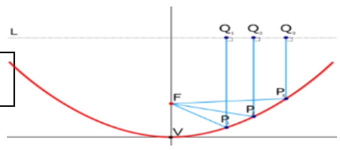
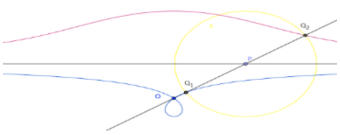
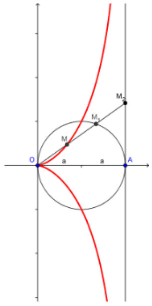
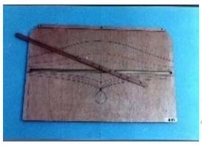






Lecture 9. Curves in the Greek Geometry, Apollonius, Great Geometer

Nicomedes 280-210BC "On conchoid lines"

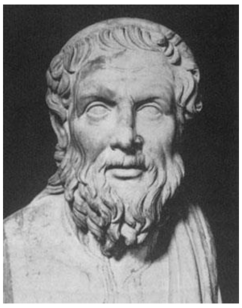


Conchoid of Nicomedes

Diocles 240-180BC "On burning mirrors" studied the focal property of parabola, cissoids of Diocles

Cissoid of Diocles

Apollonius of Perga (Ἀπολλώνιος) 262-190 BC "The Great Geometer" and astronomer. Famous

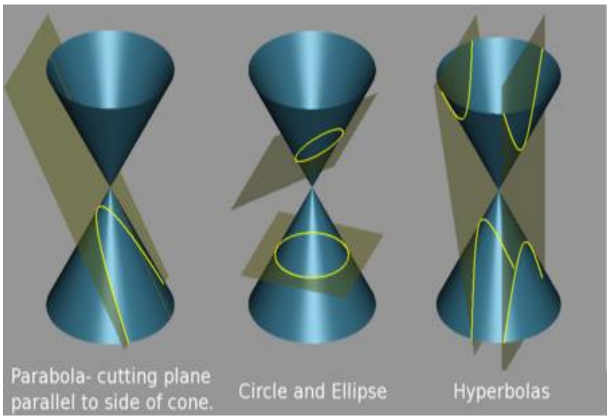


work (7 books) on conic sections where the ellipse, the parabola, and the hyperbola received their modern names. The hypothesis of eccentric orbits (deferent and epicycles) to explain the apparent motion of the planets and the varying speed of the Moon, is also attributed to him. 7 books of *Conics* (Κωνικά)

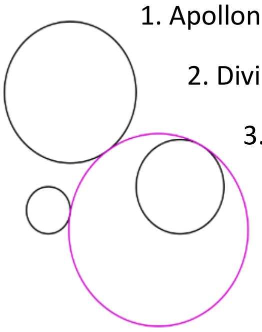
Books 1-4: elementary introduction (essential part of the results in Book 3 and all in Book 4 are original)

Book 5-7 (highly original): studies of normals, determines centers of curvature and defines evolute.

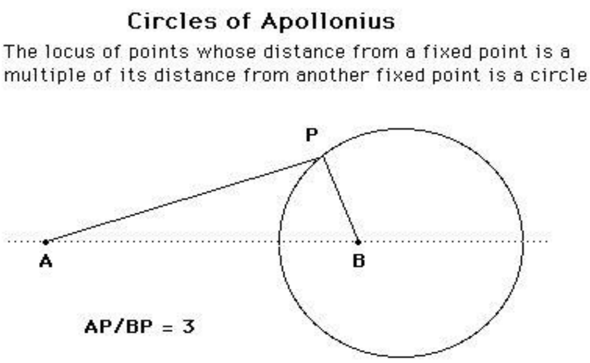
A method similar to analytic geometry is developed; difference: no negative numbers and the axis are chosen *after* coordinates are chosen depending on a given curve

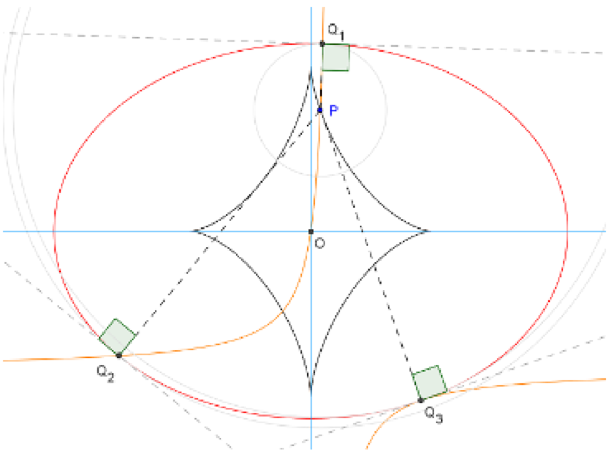


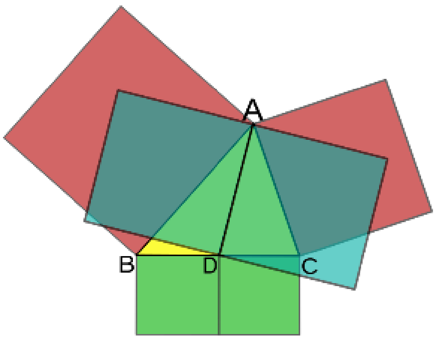
Other achievements:

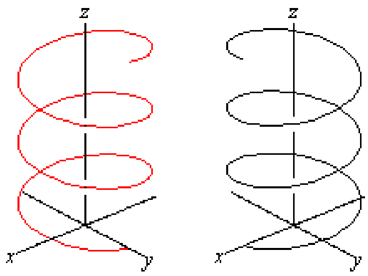


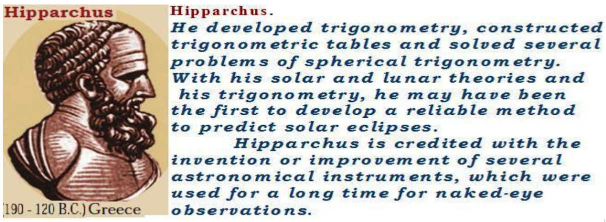
1. Apollonian definition of a circle
2. Division a line in a given ratio, harmonic section
3. Apollonian problem: construct a circle touching three things (point, lines, or circles)
4. Apollonius Theorem
5. Found the focal property of parabola
6. Studied cylindrical helix





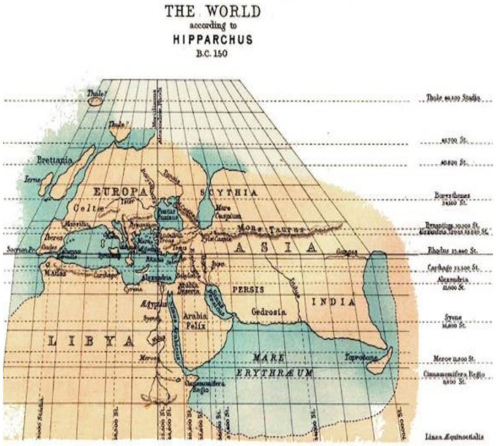




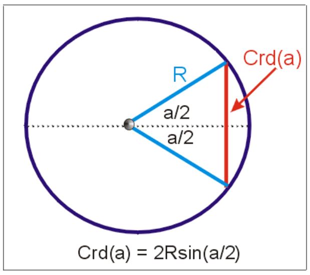


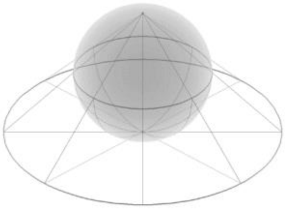
Lecture 10. Trigonometry and astronomy: Hipparchus and Ptolemy

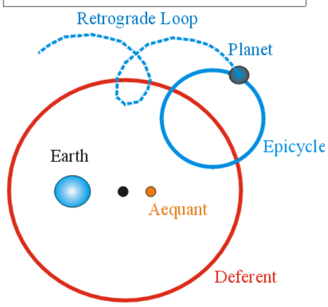
Hipparchus of Nicaea (now Isnik) 190-120BC the greatest astronomer of antiquity, also geographer and mathematician



1) A founder of trigonometry (at least he used it systematically for calculation of orbits), tabulated the values of the Chord Function (length of chard for each angle).
2) Accepted a sexagecimal full circle as 360°.
3) Transformed astronomy from purely theoretical to a practical predicative science.
4) Proved that stereographical projection is conformal (preserves angles, send circles to circles).





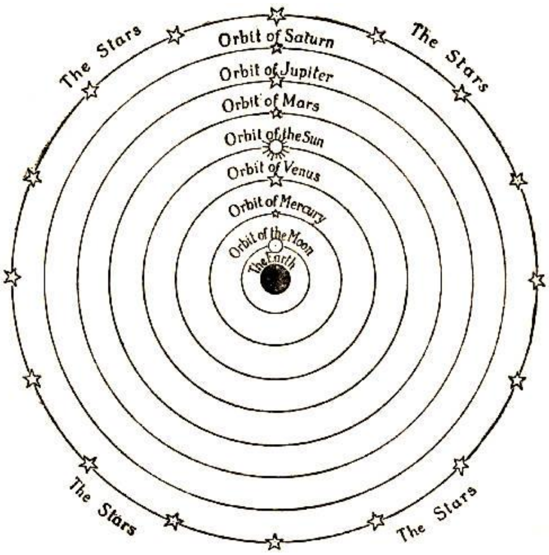


Claudius Ptolemy 90-168 AD Roman mathematician, astronomer, geographer,

worked in Alexandria, the author of Almagest (The Great Treatise) that is the only surviving treatise in astronomy, which is based generally on the works of Hipparchus. It contains a star catalogue with 48 star constellations and handy tables convenient for calculation the apparent orbits of Sun, Moon, and planets. He tried to adopt horoscopic Astrology to Aristotelean Natural Philosophy.

Ptolemy's Table of Chords

Arc (°)	$Chord_{60}$			$Chord_{10}$	$Sixtieths_{60}$				$Sixtieths_{10}$
0.5	0	31	25	0.523611	0	1	2	50	0.017226
1.0	1	2	50	1.047222	0	1	2	50	0.017226
1.5	1	34	15	1.570833	0	1	2	50	0.017226
2.0	2	5	40	2.094444	0	1	2	50	0.017226
2.5	2	37	4	2.617778	0	1	2	48	0.017226
3.0	3	8	28	3.141111	0	1	2	48	0.017226
3.5	3	39	52	3.664444	0	1	2	48	0.017226
4.0	4	11	16	4.187778	0	1	2	47	0.017226
4.5	4	42	40	4.711111	0	1	2	47	0.017226
5.0	5	14	4	5.234444	0	1	2	46	0.017226
5.5	5	45	27	5.757500	0	1	2	45	0.017226
6.0	6	16	49	6.280278	0	1	2	44	0.017226
6.5	6	48	11	6.803056	0	1	2	43	0.017226
7.0	7	19	33	7.325833	0	1	2	42	0.017225
7.5	7	50	54	7.848333	0	1	2	41	0.017225
8.0	8	22	15	8.370833	0	1	2	40	0.017225
8.5	8	53	35	8.893056	0	1	2	39	0.017225
9.0	9	24	54	9.415000	0	1	2	38	0.017225
9.5	9	56	13	9.936944	0	1	2	37	0.017225
10.0	10	27	32	10.458889	0	1	2	35	0.017225
10.5	10	58	49	10.980278	0	1	2	33	0.017225
11.0	11	30	5	11.501389	0	1	2	32	0.017225
11.5	12	1	21	12.022500	0	1	2	30	0.017225
12.0	12	32	36	12.543333	0	1	2	28	0.017224
12.5	13	3	50	13.063889	0	1	2	27	0.017224



Lecture 11. Mathematics of the Late Hellenistic Period

Hero (Heron) of Alexandria 10-70 AD mathematician and engineer-inventor

Heron's formula for the area of a triangle $A = \sqrt{s(s-a)(s-b)(s-c)}$

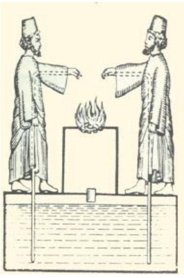
Heronian Mean (related to the volume of a truncated cone) $H = \frac{1}{3}\left(A + \sqrt{AB} + B\right).$

Heronian triangle is a triangle that has side lengths and area that are all integers (like Pytagorian ones).

In Optics: formulated the principle of the shortest path of light (stated by P.Fermat in 1662)

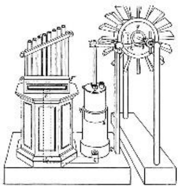
Inventions:

1) Aeolipile (steam turbine) known also as "Hero's Ball"
2) Syringe
3) Automatic temple Door opener
4) Dioptra (for geodesic measurements)



5) The first programmable robot to entertain audience at the theatre: could move in a preprogrammed way, drop metal balls, etc., for 10 minutes
6) The first vending machine to dispense holy water for coins

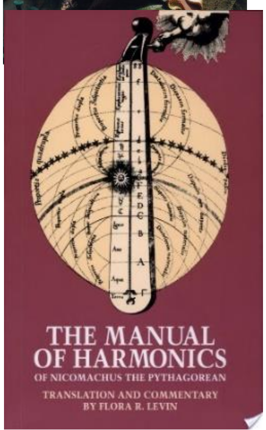
7) Fountain using sophisticated pneumatic and hydraulic principles
8) Wind powered organ (the first example of wind powered machine)



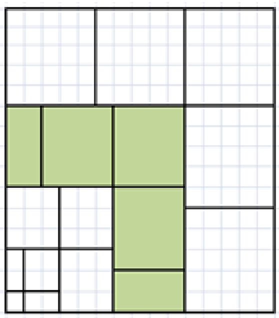
Books: *Pneumatica*, *Automata*, *Mechanica*, *Metrica*, *On the Dioptra*, *Belopoeica*, *Catoptrica*, *Geometria*, *Stereometrica*, *Mensurae*, *Cheiroballistra*, *Definitiones*, *Geodesia*, *Geoponica*

Nicomachus of Gerasa 60-120AD the author of *Introduction to Arithmetic*

where for the first time Arithmetic was separated from Geometry.
As a Neo-Pythagorean, he was interested more in some *mystical* and *divine* properties of numbers, than in conceptual and deep mathematical questions. One of his "divine" examples is an observation about cubes: $1=1^3$, $3+5=2^3$, $7+9+11=3^3$, $13+15+17+19=4^3$, etc.
Introduction to Arithmetic was a popular and influential textbook for non-mathematicians for about 1000 years, and Nicomachus was put in one row with Euclid, Pythagoras and Aristotle, although serious scholars did not respect him. Another popular book of Nicomachus was *Manual of Harmonics* based on Pythagoras and Aristotle.

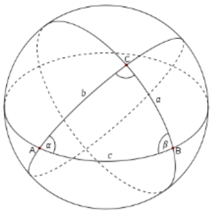


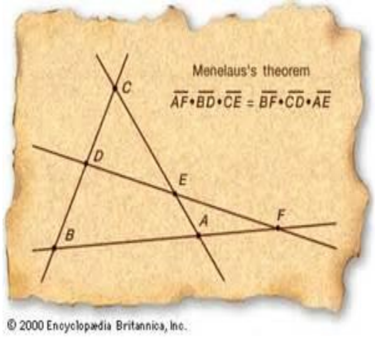
$$\left(\sum_{n=1}^{k} n\right)^2 = \sum_{n=1}^{k} n^3$$



Menelaus (Μενέλαος) of Alexandria 70 – 140 AD

mathematician and astronomer, the first to recognize geodesics on a curved surface as natural analogs of straight lines. The book Sphaerica introduces the concept of spherical triangle and proves Menelaus' theorem on collinearity of points on the edges of a triangle (which may have been previously known) and its analog for spherical triangles.





Diophantus of Alexandria (Διόφαντος) 210 -295 called "the father of algebra"

Among 13 published books just 6 survived. A treatise called *Arithmetica* deals with solving algebraic equations. Diophantine equations (including the ones known as Pell's equation and Ferma's equation) are usually algebraic equations with integer coefficients, for which integer solutions are sought. Diophantus also made advances in mathematical notation and passed to syncopated algebra from rhetorical algebra. The first one who recognized rationals as numbers and studied rational solutions of equations.

$$ax + by = 1$$

$$x^n + y^n = z^n$$

$$x^2 - ny^2 = \pm 1$$

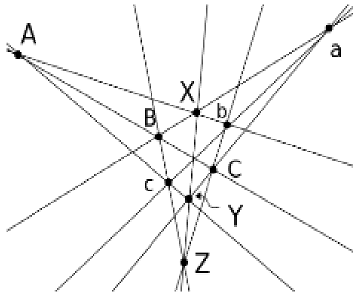
$$\frac{4}{n} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$


DIOPHANTI
ALEXANDRINI
ARITHMETICORVM
LIBRI SEX.
ET DE NVMERIS MVLTANGVLIS
LIBER VNVS.

LVTETIAE PARISIORVM,
M. DC. XXI.
CVM PRIVILEGIO REGIS.

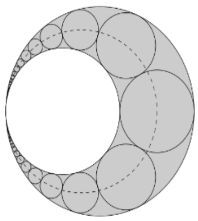
Pappus of Alexandria (Πάππος) 290–350 the last great mathematicians of Antiquity

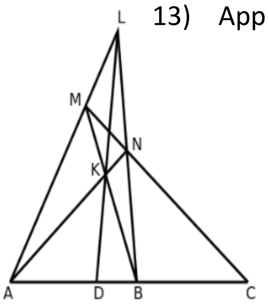
His work *Collection* (*Synagoge)* in 8 volumes mainly survived: it including geometry, recreational mathematics, doubling the cube, polygons and polyhedra. Famous for:

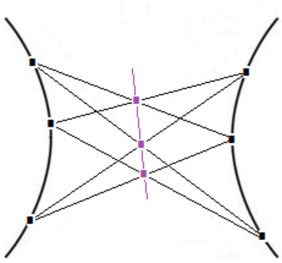


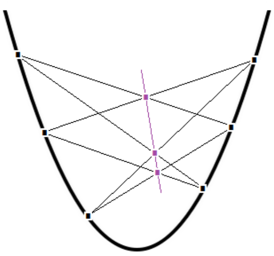

PAPPI
ALEXANDRINI
MATHEMATICAE
Collectiones.
A FEDERICO
COMMANDINO
VRBINATE
In Latinum Conuersæ, & Commentarijs
Illustratæ.
VENETIIS,
Apud Franciscum de Franciscis Senensem.
M. D. LXXXIX.

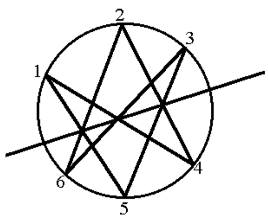
1) Pappus's Theorem in projective geometry,
2) Pappus Hexagon Theorem
3) Pappus chain of circles (centers are on an ellipse)
4) Pappus centroid (two) theorems
5) The polar line, its construction
6) Directorial property of conics
7) Perspective, the cross-ratio and harmonic ratio, complete quadrangle
8) Analyzed curves (Spirals, concoid, Quadratrix, Helix on a sphere
9) Inscribing regular polyhedra in a sphere, 13 semiregular polyhedra
10) Regular polygons, isoperimetric property
11) Explained terms "analysis" and "synthesis", "theorem", "problem", "porism"
12) Problem: how to draw an ellipse through 5 points, and similar
13) Approximate calculation of the square root of p/q.

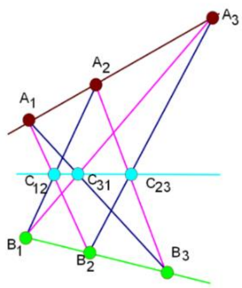


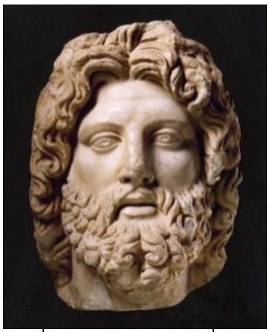












Serapis
derived from Osiris+Apis Greco-Egyptian god, patron of Ptolemaic kings

Burning of the Library of Alexandria

Column of Diocletian

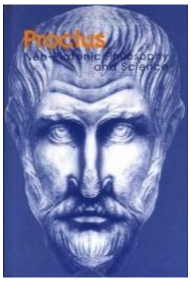
Emperor Caracalla suppressed the Musaeum in 216, and on the orders of Emperor Aurelian it was then destroyed by fire in 272. Remains of the Library of Alexandria were moved to Serapeum (temple of the god Serapis) where scholars moved the center of their studies and lectures. Roman Emperor Theodosius (after Constantine) by a decree in 380 AD forbade non-Christian worship and destructed the Temple of Apollo in Delphi (in 390 AD) and the Serapeum in Alexandria (in 391 AD). Later Diocletian erected a column at the place of destruction of Serapeum.

Theon of Alexandria 335- 405 Professor of math and astronomy in Alexandria. A competent but unoriginal mathematician famed for his commentaries on many works such as Ptolemy's *Almagest* and the works of Euclid. These commentaries were written for his students and some are even thought to be lecture notes taken by students at his lectures. He corrected mistakes which he spotted, tried to standardise Euclid's writing, and amplified Euclid's text to make it easier for beginners. Till recently, Theon's version of Euclid's *Elements* (written with the assistance of his daughter Hypatia) was the only Greek text of the *Elements*. Commented also Almagest and other classical books.

Hypatia (Υπατια) of Alexandria 370-415 math, astronomy, philosophy, inventions

Theon's daughter, the first in history woman recognized as a mathematician and scientist. She helped her father in commentaries to Euclid and Ptolemy, then commented herself to Apollonius, Diophantus, became the head of Neoplatonic School in Alexandria and taught mathematics and philosophy there.

She was kidnapped and later murdered by a Christian mob (500 monks from mountains) in a conflict between two prominent figures in Alexandria: the governor Orestes and the Bishop of Alexandria. Later (in 7th century) accused in "demonic charm" and witchery. Her murder marks downfall of Alexandrian intellectual life and the end of Classical Antiquity.



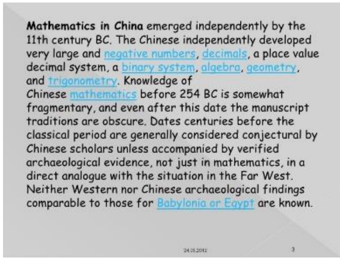
Proclus 411-485 one of the major last classical philosophers (Neoplatonist) born in Constantinople, lived and worked at Alexandria, then in Athens Became the head of Neoplatonic Academy in Athens. His commentaries on Euclid and stories about other mathematician is our principal source of knowledge about history of Greek Geometry.

Express your idea about Greek Mathematics

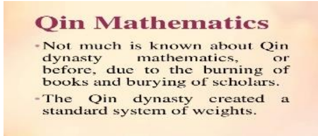
• The idea of proof. • The introduction of axioms. • Prime numbers and number theory. • Greek counting and calculation. • Greek astronomy and the impact of geometry on it. • The earliest concepts of limit.	• The concept of modeling natural phenomena on a unified basis. • Can you identify problems that could not be solved by ancient methods, but which are very near to ones that can? • Numbers: the division of valuations to length and number - a necessary consequence of incommensurables. • What were the stimuli for the particular methods and algorithms developed? • Can you identify sufficient mathematics to handle the needs of commerce?

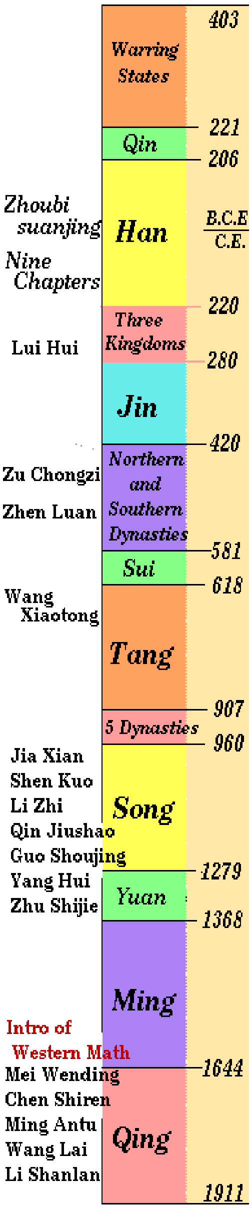
Mathematicians of Ancient Greece

• Thales of Miletus (c. 630-c 550) • Anaximander of Meletus (c. 610-c. 547) • Pythagoras of Samos (c. 570-c. 490) • Anaximenes of Miletus (fl. c. 546)) • Cleostratus of Tenedos (c. 520) • Anaxagoras of Clazomenae (c. 500-c. 428) • Zeno of Elea (c. 490-c. 430) • Antiphon of Rhamnos (the Sophist) (c. 480-411) • Oenopides of Chios (c. 450?) • Leucippus (c. 450) • Hippocrates of Chios (c. 450) • Meton (c. 430) *SB • Hippias of Elis (c. 425) • Theodorus of Cyrene (c. 425) • Socrates (469-399) • Philolaus of Croton (d. c. 390) • Democritus of Abdera (c. 460-370) • Hippasus of Metapontum (or of Sybaris or Croton) (c. 400?) • Archytas of Tarentum (of Taras) (c. 428-c. 347) • Plato (427-347) • Theaetetus of Athens (c. 415-c. 369) • Leodamas of Thasos (c. 380) • Leon (fl. c. 375) • Eudoxus of Cnidos (c. 400-c. 347) • Callipus of Cyzicus (fl. c. 370) • Xenocrates of Chalcedon (c. 396-314) • Heraclides of Pontus (c. 390-c. 322) • Bryson of Heraclea (c 350?) • Menaechmus (c. 350) • Theudius of Magnesia (c. 350?) • Thymaridas (c. 350) • Dinostratus (c. 350) • Speusippus (d. 339) • Aristotle (384-322) • Eudemus of Rhodes (the Peripatetic) (c. 335) • Autolycus of Pitane (c. 300)	• Euclid (c. 295) • Aristarchus of Samos (c. 310-230) • Archimedes of Syracuse (287-212) • Aristaeus the Elder (fl. c. 350-330) • Philo of Byzantium (fl. c. 250) • Nicoteles of Cyrene (c. 250) • Strato (c. 250) • Persius (c. 250?) • Eratosthenes of Cyrene (c. 276-c. 195) • Chrysippus (280-206) • Conon of Samos (c. 245) • Apollonius of Perga (c. 260-c. 185) • Nicomedes (c. 240?) • Dositheus of Alexandria (fl. c. 230) • Perseus (fl. 300-70 B.C.E.?) • Dionysodorus of Amisus (c. 200?) • Diocles of Carystus (c. 180) • Hypsicles of Alexandria (c. 150?) • Hipparchus of Nicaea (c. 180-c. 125) • Zenodorus (c. 100?? BCE?) • Posidonius (c. 135-c. 51) • Zeno of Sidon (c. 79 BCE) • Geminus of Rhodes (c. 77 BCE) • Cleomedes (c. 40? BCE) • Heron of Alexandria (fl. c. 62 CE) (Hero) • Theodosius of Tripoli (c. 50? CE?) • Menelaus of Alexandria (c. 100 CE) • Nicomachus of Gerasa (c. 100) • Ptolemy (Claudius Ptolemaeus) (100-178) • Diogenes Laertius (c. 200) • Diophantus of Alexandria (c. 250?) • Iamblichus (c. 250-c. 350) • Pappus of Alexandria (c. 320) • Theon of Alexandria (c. 390) • Hypatia of Alexandria (c. 370-415) • Proclus Diadochus (410-485)



Lecture 12. Mathematics of Ancient China and India



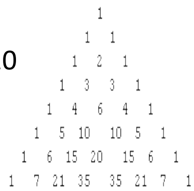


9 Chapters of the Mathematical Art fundamental work dominating the history of Chinese mathematics and playing in China the role of Euclid's Elements. It looks like a practical handbook consisting of 246 problems for practical needs: in engineering, surveying, trade and taxation. Unlike Elements 9 Chapters are not concerned about rigorous proofs.

Some Chinese (for example, Liu Hui) believe that the basis of 9 Chapters was written about 1000 BC, and later some mathematicians contributed to it. But in Qin-dynasty time (213 BC) all the copies were burned which led to the destruction of the classical knowledge. A new version is basically due to Zhang Cang in 170 BC. But mostly historians believe that 9 chapters were originally written after 200BC.

Content of the Chapters:

1) Land Surveying: 38 problems about area (triangles. Rectangles, trapeziums, circles) including addition, subtraction, multiplication and division. The Euclidean algorithm for the g.c.d. is given. Approximation of π is presented.



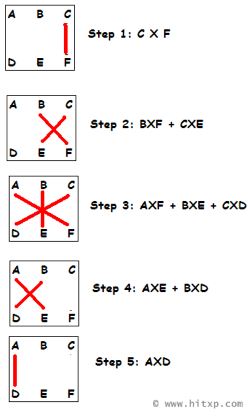
2) Millet and Rice: 46 problems about exchange of goods, with the rates of 20 different types of grains, beans and seeds. Solving proportions and percentage problems.
3) Distribution by proportion: 20 problems with direct, inverse and compound proportions. Arithmetic and geometric progression is used.
4) Short Width: some extremal problems, then square and cubic roots. Notion of limit and infinitesimal.
5) Civil Engineering: 28 problems on construction canals etc. Volumes of prisms, pyramids, wedges, cylinders. Liu Hui discussed a "method of exhaustion".
6) Fair distribution of Goods: 28 problems about ratio and proportions (travelling, taxation, sharing). Filling a cistern through 5 canals. A pursuit problem.
7) Excess and Deficit: 20 problems with the rule of double false position. Linear equations are solved by making two guesses at the solution, then finding answer from two errors.
8) Calculation by Square Tables: 18 problems with linear systems solved by Gaussian elimination (just equations are placed in columns, that is why column operations are used). Negative numbers are used.
9) Right angled triangles: 24 problems. Pythagoras theorem is known as "Gougu rule". Pythagorean triples, similar triangles. Quadratic equations are solved using a geometric square-root algorithm.

To summarize, the main goals are

1) Systematic treatment of fractions 2) Dealing with various kinds of proportions 3) Devising methods for extracting square and cubic roots 4) Solution of linear systems of equations	5) Introducing concepts of positive and negative numbers 6) Finding a formula for Pythagorean triples 7) Calculating areas and volumes of different shapes and figures

$$\pi \approx 768\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+1}}}}}}}}} \approx 3.14159046323676 3.$$

Liu Hui 263 AD one of the greatest mathematicians of ancient China

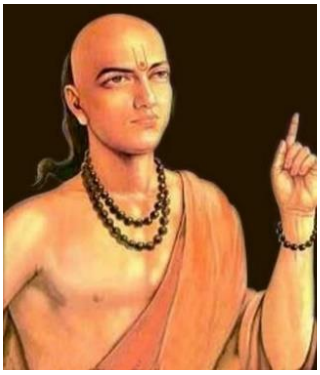


1) Used decimal fractions and obtained $\pi = 3.1416$, using a 3072-gon.
2) Prove a formula for Pythagoras triples.
3) Devised a method for solving linear systems (Gaussian elimination).
4) Solving many practical questions (finding the height, width of a river, depth, size of a city, etc.).

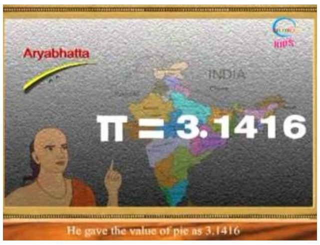
Indian Mathematics: originates in **Vedic mathematics** in Sanskrit sutras with multiplication rules and formulas (like areas of geometric figures, may be Pythagoras Th) hidden between the Vedic hymns.

Aryabhata 476-550 AD book Aryabhatiya (on Astronomy) in 121 verses

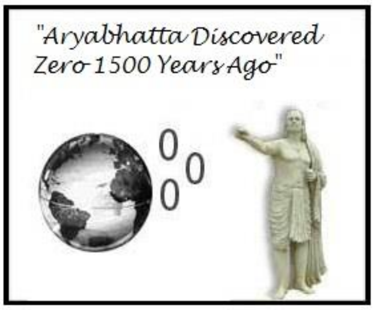
1) The place-value system; he did not use zero, but some argue that its knowledge was implicit



2) Approximation of π = 3.1416; he had possibly a guess that π is an irrational number
3) The oldest use of alphabet numerals in place of old-style word numerals

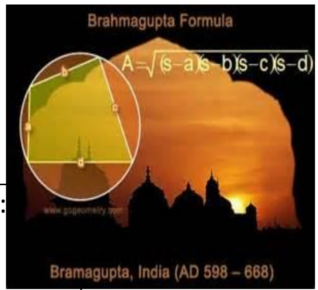


4) Used arithmetic and Geometric progressions
5) Used trigonometry for the computation of eclipses

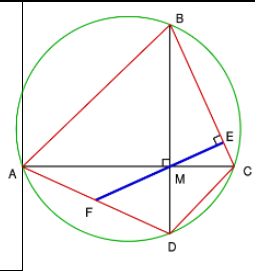


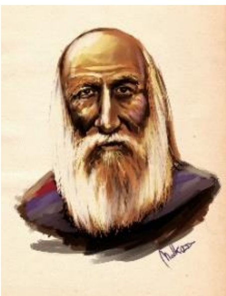
BRAMHAGUPTA (598AD-668AD)

Brahmagupta 598- 668



-Solution of quadratic equation -Solution of systems by elimination method -Syncopated algebra -Sum of squares and cubes of the first n integers -Zero as a number is mentioned for the first time in his book; operations with zero are described (a problem with division)	-Positive and negative numbers: properties of their addition and multiplication -Found Pythagorean triples -Pell's equation $Nx^2+1=y^2$ -Brahmagupta theorem on inscribed quadrilateral with perpendicular diagonals -Formula for the area A= $\sqrt{(t-p)(t-q)(t-r)(t-s)}$.



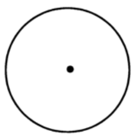


Bhāskara I 600–680

Bhāskara I 600–680 was the first to write numbers in the Hindu decimal system, with a circle for the zero. He gave a rational approximation of the sine function in commentary on Aryabhata's work.

Bhāskara II 1114–1185 worked in astronomy, calculus, algebra, and spherical trigonometry.

The ancient Hindu symbol of a circle with a dot in the middle, known as *bindu* or *bindhu*, symbolizing the void and the negation of the self, was probably instrumental in the use of a circle as a representation of the concept of zero.

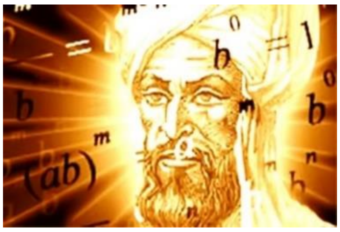


Lecture 13. Mathematics of Islamic Middle East

House of Wisdom founded by the Abbasid Caliph Abu Ja'far Al Mansour in the 8th century, for translating foreign books. Many foreign works were translated into Arabic from Greek, Chinese, Sanskrit, Persian and Syriac. Scholars produced important original research. New discoveries motivated revised translations that commented, corrected or added to the work of ancient authors. Ptolemy's Almagest, is an Arabic modification of the original name of the work: Megale Syntaxis.

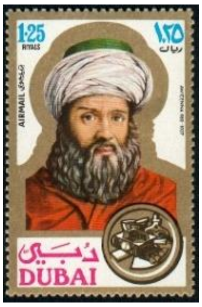
Muḥammad ibn Mūsā al-Khwārizmī- 780 – 850 (Algoritmi) "father of algebra" a Persian mathematician, astronomer and geographer during the Abbasid Caliphate, a scholar in the House of Wisdom in Baghdad

His systematic approach in solving linear and quadratic equation, leaded to algebra, a word derived from the title of his 830 book on the subject, *The Compendious Book on Calculation by Completion and Balancing* (*ḥisāb al-jabr wal-muqābala*). Algebra was a unifying theory which allowed rational numbers, irrational numbers, geometrical magnitudes, etc., to all be treated as "algebraic objects". It gave mathematics a whole new development path.

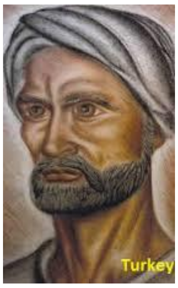


Another book, *On the Calculation with Hindu Numerals* written about 825, was principally responsible for spreading the Hindu–Arabic numeral system throughout the Middle East and Europe. It was translated into Latin as *Algoritmi de numero Indorum*. Al-Khwārizmī, rendered as (Latin) *Algoritmi*, led to the term algorithm. He assisted a project to determine the circumference of the Earth and in making a world map.

When, in the twelfth century, his works spread to Europe through Latin translations, it had a profound impact on the advance of mathematics in Europe.



Al-Ṣābiʾ Thābit ibn Qurra al-Ḥarrānī 826–901 Arabic mathematician, physician, astronomer, and translator of the Islamic Golden Age who lived in Baghdad. In mathematics, Thabit discovered an equation for determining amicable numbers. He is known for having calculated the solution to a chessboard problem involving an exponential series. He also described a Pythagoras theorem.

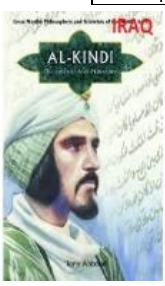


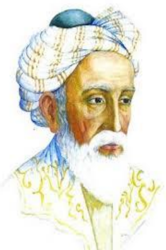
ʿAbd al-Hamīd ibn Turk (830), Turkic mathematician, the author of Logical Necessities in Mixed Equations, which is similar to al-Khwarzimi's *Al-Jabr* and was published at around the same time as, or even possibly earlier than, *Al-Jabr*. The manuscript gives exactly the same geometric demonstration as is found in *Al-Jabr*, and in one case the same example as found in *Al-Jabr*, and even goes beyond *Al-Jabr* by giving a geometric proof that if the discriminant is negative then the quadratic equation has no solution.

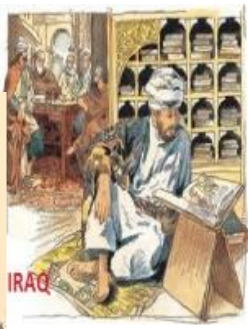
Al Hajjaj ibn Yusuf ibn Mattar ?-833
the first translator of Eucid's *Elements*

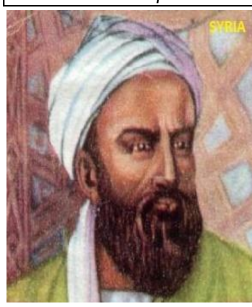
Al-Abbas ibs Saed ibn Jawhari *Commentary on Euclid's Elements,* 50 new propositions, attempts to prove 5th postulate

Fateh al Harrani al Hasib ?-825
Addition and Separation, The cubes, Fundamentals of Arithmetic

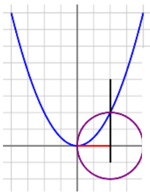


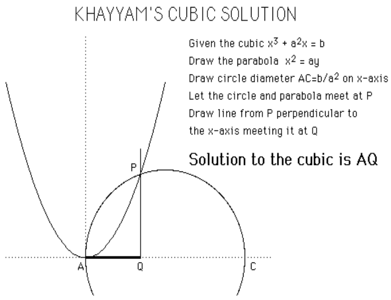


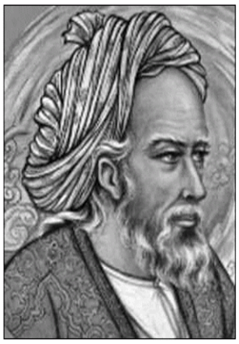




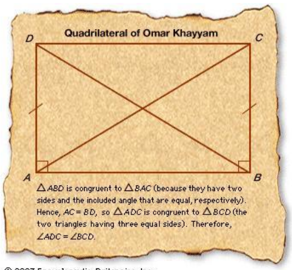
Omar Khayyám (1038-1123) Persian mathematician, astronomer, philosopher, and poet, who is widely considered to be one of the most influential scientists of all time.







Omar Khayyám (1038-1123) Persian mathematician, astronomer, philosopher, and poet, who is widely considered to be one of the most influential scientists of all time. He wrote numerous treatises on mechanics, geography, mineralogy and astrology. His *Treatise on Demonstration of Problems of Algebra* containing a systematic solution of third-degree equations, going beyond the *Algebra* of Khwārazmī, Khayyám obtained the solutions of these equations by finding the intersection points of two conic sections. This method had been used by the Greeks, but they did not generalize the method to cover all equations with positive roots. A book *Explanations of the difficulties in the postulates in Euclid's Elements* where he analyzed the parallel postulate contributed to the development of non-Euclidean geometry.



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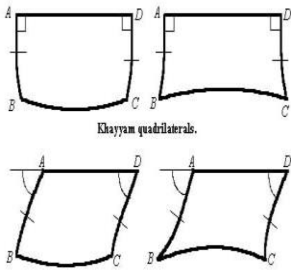
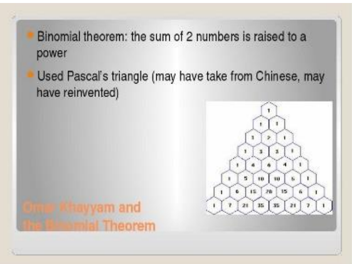
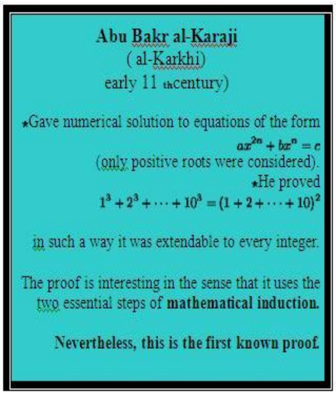




In considering Pascal's triangle, known in Persia as "Khayyam's triangle".

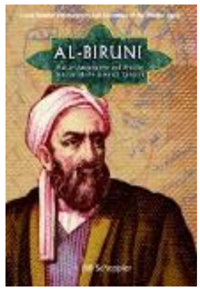
Abū Bakr ibn Muḥammad ibn al Ḥusayn al-Karajī (or al-Karkhī) 953 –1029 mathematician and engineer working at Baghdad.

1) His work on algebra and polynomials gave the rules for arithmetic operations for adding, subtracting and multiplying polynomials; though he was restricted to dividing polynomials by monomials.
2) He systematically studied the algebra of exponents, and was the first to realize that the sequence $x, x^2, x^3,\ldots$ and the reciprocals $1/x, 1/x^2, 1/x^3,\ldots$ could be extended indefinitely.
3) He wrote on the binomial theorem and Pascal's triangle.
4) In a now lost work, he introduced the idea of argument by mathematical induction.



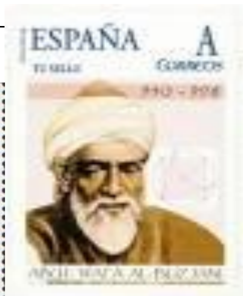
Sharaf al-Dīn al-Ṭūsī 1135-1213 Persian mathematician and astronomer.

In treatise on cubic equation analyzed the maximum of $y=bx-x^3$ by taking the derivative. Then Al-Tusi deduces that the equation has a positive root if $D = b^3/27 - a^2/4 \geq 0$, where D is the discriminant of the equation. Al-Tusi then went on to give what we would essentially call the Ruffini-Horner method for approximating the root of the cubic equation. A treatise on the two lines that approach each other, but never intersect.



Ahmed bin Muhannad al Hasib ?-927 book *Addition and Separation*

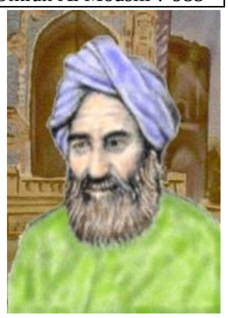
Abu Al Vafa Al Buzajani ?-1000 sperical trigonometry, book using negative numbers, introduced sec, cosec

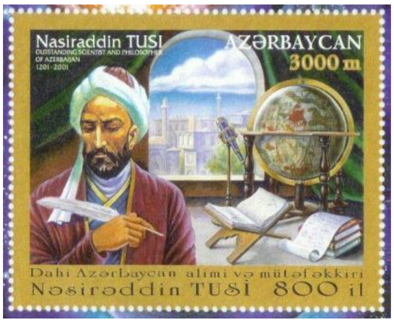


Abu Ali Al Hasan ibn Al Haytham (Al Hazen) ?-1039 elliptic and hyperbolic geometry

Abu Yousuf Yaqub Al Razi

Ali bun Ahmet bin Omran Al Mousili ?-955





Nasir al-Din al-Tusi 1201-1274 Persian polymath and prolific writer: an architect, astronomer, biologist, chemist, mathematician, philosopher, physician, physicist, scientist, theologian. Perhaps, the greatest of the later Persian scholars.

Main works in Mathematics:

Book on the complete quadrilateral: A five volume summary of trigonometry including spherical trigonometry. This is the first treatise on trigonometry independent of astronomy.

On the Sector Figure, appears the famous law of sines for plane triangles (stated also for spherical triangles), the law of tangents for spherical triangles, including the proofs.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Al-Kāshī 1380–1429 Persian astronomer and mathematician

In French, the law of cosines is named Theorem of Al-Kashi, as al-Kashi was the first to provide its explicit statement.

In *The Treatise on the Chord and Sine*, al-Kashi computed sin 1° to nearly as much accuracy as his value for π, which was the most accurate approximation of sin 1° in his time. In algebra and numerical analysis, he developed an iterative method for solving cubic equations, which was not discovered in Europe until centuries later.

A method algebraically equivalent to Newton's method was known to his predecessor Sharaf al-Dīn al-Tūsī. Al-Kāshī improved on this by using a form of Newton's method to solve $x^P - N = 0$ to find roots of *N*.

In order to determine sin 1°, al-Kashi discovered the following formula often attributed to François Viète in the 16th century:

$$\sin 3\phi = 3\sin\phi - 4\sin^3\phi$$

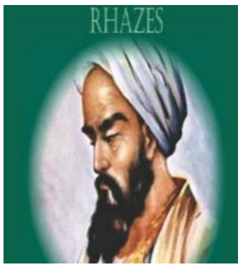
He correctly computed 2π to 9 sexagesimal digits in 1424, and he converted this approximation of 2π to 17 decimal places of accuracy. Al-Kashi's goal was to compute the circle constant so precisely that the circumference of the largest possible circle (ecliptics) could be computed with highest desirable precision (the diameter of a hair).

Al Samaw'al ibn al-Maghribi 1130-1180, book at age 19 approximation of n-th root

Kamāl al-Dīn al-Fārisī 1267-1319, impossibility of solution $x^4+y^4=z^4$, amicable numbers

Abu Bakr al-Hassar 12th century invented modern notation for fractions

Al Murrakishi 1258-1321



Timeline of (some) Math Events till 13th Century

Date	Name	Nationality	Major Achievements
35000 BC		African	First notched tally bones
3100 BC		Sumerian	Earliest documented counting and measuring system
2700 BC		Egyptian	Earliest fully-developed base 10 number system in use
2600 BC		Sumerian	Multiplication tables, geometrical exercises and division problems
2000-1800 BC		Egyptian	Earliest papyri showing numeration system and basic arithmetic
1800-1600 BC		Babylonian	Clay tablets dealing with fractions, algebra and equations
1650 BC		Egyptian	Rhind Papyrus (instruction manual in arithmetic, geometry, unit fractions, etc)
1200 BC		Chinese	First decimal numeration system with place value concept
1200-900 BC		Indian	Early Vedic mantras invoke powers of ten from a hundred all the way up to a trillion
800-400 BC		Indian	"Sulba Sutra" lists several Pythagorean triples and simplified Pythagorean theorem for the sides of a square and a rectangle, quite accurate approximation to √2
650 BC		Chinese	Lo Shu order three (3 x 3) "magic square" in which each row, column and diagonal sums to 15
624-546 BC	Thales	Greek	Early developments in geometry, including work on similar and right triangles
570-495 BC	Pythagoras	Greek	Expansion of geometry, rigorous approach building from first principles, square and triangular numbers, Pythagoras' theorem
500 BC	Hippasus	Greek	Discovered potential existence of irrational numbers while trying to calculate the value of √2
490-430 BC	Zeno of Elea	Greek	Describes a series of paradoxes concerning infinity and infinitesimals
470-410 BC	Hippocrates of Chios	Greek	First systematic compilation of geometrical knowledge, Lune of Hippocrates
460-370 BC	Democritus	Greek	Developments in geometry and fractions, volume of a cone
428-348 BC	Plato	Greek	Platonic solids, statement of the Three Classical Problems, influential teacher and popularizer of mathematics, insistence on rigorous proof and logical methods
410-355 BC	Eudoxus of Cnidus	Greek	Method for rigorously proving statements about areas and volumes by successive approximations
384-322 BC	Aristotle	Greek	Development and standardization of logic (although not then considered part of mathematics) and deductive reasoning
300 BC	Euclid	Greek	Definitive statement of classical (Euclidean) geometry, use of axioms and postulates, many formulas, proofs and theorems including Euclid's Theorem on infinitude of primes
287-212 BC	Archimedes	Greek	Formulas for areas of regular shapes, "method of exhaustion" for approximating areas and value of π, comparison of infinities
276-195 BC	Eratosthenes	Greek	"Sieve of Eratosthenes" method for identifying prime numbers
262-190 BC	Apollonius of Perga	Greek	Work on geometry, especially on cones and conic sections (ellipse, parabola, hyperbola)
200 BC		Chinese	"Nine Chapters on the Mathematical Art", including guide to how to solve equations using sophisticated matrix-based methods
190-120 BC	Hipparchus	Greek	Develop first detailed trigonometry tables
36 BC		Mayan	Pre-classic Mayans developed the concept of zero by at least this time
10-70 AD	Heron (or Hero) of Alexandria	Greek	Heron's Formula for finding the area of a triangle from its side lengths, Heron's Method for iteratively computing a square root
90-168 AD	Ptolemy	Greek/Egyptian	Develop even more detailed trigonometry tables

200 AD	Sun Tzu	Chinese	First definitive statement of Chinese Remainder Theorem
200 AD		Indian	Refined and perfected decimal place value number system
200-284 AD	Diophantus	Greek	Diophantine Analysis of complex algebraic problems, to find rational solutions to equations with several unknowns
220-280 AD	Liu Hui	Chinese	Solved linear equations using a matrices (similar to Gaussian elimination), leaving roots unevaluated, calculated value of π correct to five decimal places, early forms of integral and differential calculus
400 AD		Indian	"Surya Siddhanta" contains roots of modern trigonometry, including first real use of sines, cosines, inverse sines, tangents and secants
476-550 AD	Aryabhata	Indian	Definitions of trigonometric functions, complete and accurate sine and versine tables, solutions to simultaneous quadratic equations, accurate approximation for π (and recognition that π is an irrational number)
598-668 AD	Brahmagupta	Indian	Basic mathematical rules for dealing with zero (+, - and x), negative numbers, negative roots of quadratic equations, solution of quadratic equations with two unknowns
600-680 AD	Bhaskara I	Indian	First to write numbers in Hindu-Arabic decimal system with a circle for zero, remarkably accurate approximation of the sine function
780-850 AD	Muhammad Al-Khwarizmi	Persian	Advocacy of the Hindu numerals 1 - 9 and 0 in Islamic world, foundations of modern algebra, including algebraic methods of "reduction" and "balancing", solution of polynomial equations up to second degree
908-946 AD	Ibrahim ibn Sinan	Arabic	Continued Archimedes' investigations of areas and volumes, tangents to a circle
953-1029 AD	Muhammad Al-Karaji	Persian	First use of proof by mathematical induction, including to prove the binomial theorem
966-1059 AD	Ibn al-Haytham (Alhazen)	Persian/Arabic	Derived a formula for the sum of fourth powers using a readily generalizable method, "Alhazen's problem", established beginnings of link between algebra and geometry
1048-1131	Omar Khayyam	Persian	Generalized Indian methods for extracting square and cube roots to include fourth, fifth and higher roots, noted existence of different sorts of cubic equations
1114-1185	Bhaskara II	Indian	Established that dividing by zero yields infinity, found solutions to quadratic, cubic and quartic equations (including negative and irrational solutions) and to second order Diophantine equations, introduced some preliminary concepts of calculus
1170-1250	Leonardo of Pisa (Fibonacci)	Italian	Fibonacci Sequence of numbers, advocacy of the use of the Hindu-Arabic numeral system in Europe, Fibonacci's identity (product of two sums of two squares is itself a sum of two squares)
1201-1274	Nasir al-Din al-Tusi	Persian	Developed field of spherical trigonometry, formulated law of sines for plane triangles
1202-1261	Qin Jiushao	Chinese	Solutions to quadratic, cubic and higher power equations using a method of repeated approximations
1238-1298	Yang Hui	Chinese	Culmination of Chinese "magic" squares, circles and triangles, Yang Hui's Triangle (earlier version of Pascal's Triangle of binomial co-efficients)